

$T$-fuzzy $TL$-ideal of $\Gamma$-near ring

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Abstract

In this paper, we establish and revise the concept of $T$-fuzzy $TL$-ideal of $\Gamma$-near ring. Also the notations of $TL$-ideal of $\Gamma$-near ring were introduced with some related properties.

Keywords

$\Gamma$-near ring, $TL$-ideal, $TL$-Fuzzy ideal, Direct Product of $TL$- Fuzzy Sub $\Gamma$- near ring, Quotient $\Gamma$-near ring with $\ast$-norm.

AMS Subject Classification

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1. Introduction

The theory of fuzzy sets and their related properties was introduced by Zadeh.L.A [15] in 1965. In 1991, Abou Zaid. S [1] defined a Fuzzy subnear-rings and ideals. In 1996, Seung dong kim and hee sikkim was defined as the homomorphic image of fuzzy ideals and some related properties. The notation of $\Gamma$-near ring was introduced by Bhavanari Satyanarayana and Syam Prasad. K [11]. In 2007, Akram. M was introduced the $T$-fuzzy ideals in near rings. In 2012, Srinivas.T and Nagaiah.T [13] was presented $T$-fuzzy ideal of $\Gamma$-near ring has several properties of $\Gamma$-near rings. In this paper, by using $T$- fuzzy ideal and $TL$-ideal of $\Gamma$-near ring all the above are use them. Further, additionally we introduce homomorphic images and direct product of $T$-fuzzy $TL$-ideal of $\Gamma$-near ring. We may expand to this paper as the $\Gamma$- near ring from a theoretical portion.

2. Preliminaries

In this section, we review the some definitions that will be required in this paper.

Definition 2.1. A non empty set $N$ with two binary operations $\ast$ and $\cdot$ is called a near ring if it satisfies the following axioms:

(i) $(N, \ast)$ is a group.

(ii) $(N, \cdot)$ is a semi group.

(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ for all $x, y, z \in N$.

Precisely speaking it is a left near ring because it satisfies the left distributive law. We will use the word near ring instead of “Left near-ring”. We denote $xy$ instead of $x \cdot y$. Note that $x0 = 0$ and $x(-y) = -xy$, but $0x \neq 0$ for $x, y \in N$.

Definition 2.2. Let $(R, +)$ be a group and $\Gamma$ be a nonempty set. Then $R$ is said to be a $\Gamma$-near ring if their exist a mapping $R \times \Gamma \times R \rightarrow R$ satisfies the following conditions:

(i) $(x+y)\alpha z = x\alpha z + y\alpha z$.

(ii) $(x\alpha y)\beta z = x\alpha (y\beta z)$

for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$.

Definition 2.3. Let $R$ be a $\Gamma$- near ring. A normal subgroup $(I, +)$ of $(R, +)$ is called

(i) a left ideal if $x\alpha(y+i) = x\alpha y \in I$ for all $x, y \in R, \alpha \in \Gamma, i \in I$.

(ii) a right ideal if $i\alpha x \in I$ for all $x \in R, \alpha \in \Gamma, i \in I$

(iii) an ideal if it is both a left ideal and a right ideal of $R$.

A $\Gamma$- near ring $R$ is said to be a zero – symmetric if $a\alpha 0 = 0$ for all $a \in R$ and $\alpha \in \Gamma$, where 0 is additive identity in $R$.

Contents

1 Introduction ............................................. 206
2 Preliminaries ........................................... 206
3 $T$-fuzzy $TL$-ideal of $\Gamma$-near ring .................. 207

References ............................................. 211
Definition 2.4. A subset $M$ of a $\Gamma$-near ring $R$ is said to be a sub $\Gamma$-near ring if there exist a mapping $M \times \Gamma \times M \to M$ such that

(i) $(M,+)$ be a subgroup of $(R,+)$.
(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$ for every $x,y,z \in M$ and $\alpha \in \Gamma$.
(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for every $x,y,z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.5. A fuzzy sub near ring $A$ of $R$ is called a fuzzy ideal if it satisfies the following conditions:

(i) $A(y+x-y) \geq A(x)$ for all $x,y,z \in R$.
(ii) $A(xy) \geq A(y)$ for all $x,y \in R$.
(iii) $A((x+i)y-xy) \geq A(i)$ for all $x,y,i \in R$.

Definition 2.6. A binary operation $T$ on a lattice $L$ is called a $\Gamma$-fuzzy $T$-L-ideal of $R$ if it satisfies the following conditions:

(i) $A(y+x-y) \geq A(x)$ for all $x,y,z \in R$.
(ii) $A(xy) \geq A(y)$ for all $x,y \in R$.
(iii) $A((x+i)y-xy) \geq A(i)$ for all $x,y,i \in R$.

Definition 2.7. Let $\rho$ be a fuzzy $T$-fuzzy $T$-L-ideal of $R$ then $\rho = \lambda \circ f$ in $R$. Similarly, if $\lambda$ is a fuzzy set in $f(R)$, then $\rho = \lambda \circ f$ in $R$ is defined as $\rho(x) = \lambda(f(x))$ for all $x \in R$ and is called the pre image of $\lambda$ under $f$.

3. $T$-fuzzy $T$-L-ideal of $\Gamma$-near ring

Let $R$ be a near-ring and $L$ be a complete lattice.

Theorem 3.1. If $L$-subset of $A \in L^R$ is a $T$-fuzzy $T$-L-ideal of $R$ then, $A(x+y) \geq A(x)$ for all $x,y \in R$.

Proof. Let $L$-subset of $A$ be a $T$-fuzzy $T$-L-ideal of $R$ then we have, $A(x+y) \geq A(0)$. But, $A(0) \leq A(x)$. Thus, $A(x-y) = A(0)$. Now consider,

$$A(x) = A(y+x-y) = A(y+(x-y)) \geq A(x)TA(y) \geq A(y).$$

Similarly we can prove that $A(y) \geq A(x)$. Hence $A(x) = A(y)$ for all $x,y \in R$. 


Theorem 3.2. If $L$- subset of $A$ and $B \in L^R$ are $T$-fuzzy $TL$-ideal of a $\Gamma$- near ring $R$ then $A \cap B$ is a $T$-fuzzy $TL$-ideal of $R$.

Proof. (i) \[(A \cap B)(0) \geq A(0)TB(0) = 1\]

(ii) \[(A \cap B)(-x) \geq A(-x)TB(-x) \geq A(0-x)TB(0-x) \geq A(0)TA(x)TB(0)TB(x) \geq A(x)TB(x) \geq (A \cap B)(x)\]

(iii) \[(A \cap B)(x-y) \geq A(x-y)TB(x-y) \geq A(x)TA(y)TB(x)TB(y) \geq (A(x)TB(x))(A(y)TB(y)) \geq (A \cap B)(x)TA(A \cap B)(y)\]

(iv) \[(A \cap B)(y+x-y) \geq A(y+x-y)TB(y+x-y) \geq A(x)TB(x) \geq (A \cap B)(x)\]

(v) \[(A \cap B)(x\alpha y) \geq A(x\alpha y)TB(x\alpha y) \geq A(y)TB(y) \geq (A \cap B)(y)\]

(vi) \[(A \cap B)[(x\alpha(z+y)-x\alpha y)] \geq A[(x\alpha(z+y)-x\alpha y)]TB[(x\alpha(z+y)-x\alpha y)] \geq A(z)TB(z) \geq (A \cap B)(z).\]

Hence $A \cap B$ is a $T$-fuzzy $TL$-ideal of $R$. This completes the proof.

Theorem 3.3. Let $L$- subset of $A \in L^R$ be a $T$-fuzzy $TL$-ideal of a $\Gamma$- near ring $R$ and $L$- subset of $A^* \in L^R$ be a fuzzy set in $R$ then $A^*$ defined by, $A^* = \frac{A(x)}{A(1)} \forall x \in R$. Then $A^*$ is a normal $T$-fuzzy $TL$-ideal of $R$ contains $A$.

Proof. Let $L$- subset of $A \in L^R$ be a $T$-fuzzy $TL$-ideal of a $\Gamma$- near ring $R$. For any $x, y, z \in R$ and $\alpha \in \Gamma$, we have,

(i) \[A^*(0) = \frac{A(0)}{A(1)} \geq 1\]

(ii) \[A^*(-x) = \frac{A(-x)}{A(1)} \geq \frac{A(0-x)}{A(1)} \geq \frac{A(0)TA(x)}{A(1)} \geq \frac{A(x)}{A(1)} \geq A^*(x)\]

(iii) \[A^*(x-y) = \frac{A(x-y)}{A(1)} \geq \frac{A(x)TA(y)}{A(1)} \geq \frac{A(x)A(1)TA^*(y)}{A(1)} \geq A^*(x)TA^*(y)\]

(iv) \[A^*(y+x-y) = \frac{A(y+x-y)}{A(1)} \geq \frac{A(x)}{A(1)} \geq A^*(x)\]

(v) \[A^*(x\alpha y) = \frac{A(x\alpha y)}{A(1)} \geq \frac{A(y)}{A(1)} \geq A^*(y)\]

(iv) \[A^*[x\alpha(z+y)-x\alpha y] = \frac{A[x\alpha(z+y)-x\alpha y]}{A(1)} \geq \frac{A(z)}{A(1)} \geq A^*(z)\]

Hence $A^*$ is a normal $T$-fuzzy $TL$-ideal of $R$ contains $A$. 

208
Theorem 3.4. Let \( L \)-subset of \( A \in \mathbb{L}_R \) be a \( T \)-fuzzy \( TL \)-ideal of a \( \Gamma \)-near ring \( R \) and let \( L \)-subset of \( A^+ \in \mathbb{L}^R \) be a fuzzy set in \( R \) then \( A^+ \) is defined by, \( A^+(x) = A(x) + 1 + A(1) \) for all \( x \in R \). Then \( A^+ \) is a \( T \)-fuzzy \( TL \)-ideal of \( \Gamma \)-near ring \( R \) containing \( A \).

Proof. Let \( L \)-subset of \( A \in \mathbb{L}_R \) be a \( T \)-fuzzy \( TL \)-ideal of a \( \Gamma \)-near ring \( R \). For any \( x, y, z \in R \) and \( \alpha \in \Gamma \).

(i) \[ A^+(0) = A(0) + 1 + A(1) = 1. \]

(ii) \[
A^+(-x) = A(0 - x) + 1 + A(1) \\
\geq (A(0) + 1 + A(1))TA(x) + 1 + A(1) \\
\geq A(x) + 1 + A(1) \\
\geq A^+(x)
\]

(iii) \[
A^+(x - y) = A(x - y) + 1 + A(1) \\
\geq (A(x) + 1 + A(1))TA(y) + 1 + A(1) \\
\geq A^+(x)TA^+(y).
\]

(iv) \[
A^+(y + x - y) = A(y + x - y) + 1 + A(1) \\
\geq A(x) + 1 + A(1) \\
\geq A^+(x).
\]

(v) \[
A^+(x\alpha y) = A(x\alpha y) + 1 + A(1) \\
\geq A(y) + 1 + A(1) \\
\geq A^+(y).
\]

(vi) \[
A^+[x\alpha(z + y) - x\alpha y] = A(x\alpha(z + y) - x\alpha y) + 1 + A(1) \\
\geq A(z) + 1 + A(1) \\
\geq A(z).
\]

Hence \( A^+ \) is a \( T \)-fuzzy \( TL \)-ideal of \( \Gamma \)-near ring \( R \) containing \( A \).

\[ \square \]

Theorem 3.5. An onto homomorphic image of a \( T \)-fuzzy \( TL \)-ideal with sup property is a \( T \)-fuzzy \( TL \)-ideal.

Proof. Let \( M \) and \( N \) are \( \Gamma \)-near rings. Let \( f : M \rightarrow N \) be epimorphism and \( L \)-subset of \( A \in L^R \) be a \( T \)-fuzzy \( TL \)-ideal of \( R \) with sup property. Let \( x, y \in N, x_0 \in f'(x), y_0 \in f'(y) \) and \( z_0 \in f'(z) \) be such that \( A(x_0) = \sup_{n \in f'(x)} A(n), A(y_0) = \sup_{n \in f'(y)} A(n), A(z_0) = \sup_{n \in f'(z)} A(n) \) respectively. Then for any \( \alpha \in A, \) we have,

(i) \[ A^f(0) = \sup_{z \in f'(0)} A(z) \geq A(0) = 1. \]

(ii) \[
A^f(-x) = \sup_{z \in f'(-x)} A(z) \\
\geq A(x_0) \\
\geq \sup_{n \in f'(x)} A(n) \\
= A^f(x).
\]

(iii) \[
A^f(x - y) = \sup_{z \in f'(x - y)} A(z) \\
\geq A(x_0 - y_0) \\
\geq \sup_{n \in f'(x)} A(n) \sup_{n \in f'(y)} A(n) \\
= A^f(y).
\]

(iv) \[
A^f(y + x - y) = \sup_{z \in f'(y + x - y)} A(z) \\
\geq A(x_0) \\
\geq \sup_{n \in f'(x)} A(n) \\
= A^f(x).
\]

(v) \[
A^f(x\alpha y) = \sup_{z \in f'(x\alpha y)} A(z) \\
\geq A(y_0) \\
\geq \sup_{n \in f'(y)} A(n) \\
= A^f(y).
\]

(vi) \[
A^f[x\alpha(z + y) - x\alpha y] = \sup_{z \in f'(x\alpha y)} A(z) \\
\geq A(y_0) \\
\geq \sup_{n \in f'(y)} A(n) \\
= A^f(y).
\]
Theorem 3.6. An epimorphic pre image of a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring is a $T$-fuzzy $TL$-ideal of $R$.

Proof. Let $M$ and $N$ be $\Gamma$ near rings. Let $f: M \to N$ is an epimorphism. Let $L$-subset of $\lambda \in L^R$ be the $T$-fuzzy $TL$-ideal of $N$ and $\rho$ be the pre image of $\lambda$ under $f$. Then for any $x, y, z \in M$ and $\alpha \in \Gamma$. We have,

(i) $\rho(0) = (\lambda \circ f)(0) = \lambda(f(0)) = 1.$

(ii) $
\begin{align*}
\rho(-x) &= (\lambda \circ f)(-x) \\
&= \lambda(f(-x)) \\
&\geq \lambda(f(x)) \\
&\geq (\lambda \circ f)(x) \\
&\geq \rho(x)
\end{align*}
$

(iii) $
\begin{align*}
\rho(x - y) &= (\lambda \circ f)(x - y) \\
&= \lambda(f(x - y)) \\
&\geq \lambda(f(x))T(f(y)) \\
&\geq \lambda(f(x))T(\lambda(f(y)) \\
&\geq (\lambda \circ f)(x)T(\lambda \circ f)(y) \\
&\geq \rho(x)\rho(y).
\end{align*}
$

(iv) $
\begin{align*}
\rho(y + x - y) &= (\lambda \circ f)(y + x - y) \\
&= \lambda(f(y + x - y)) \\
&\geq \lambda(f(x)) \\
&\geq (\lambda \circ f)(x) \\
&\geq \rho(x).
\end{align*}
$

(v) $
\begin{align*}
\lambda(x\alpha y) &= (\lambda \circ f)(x\alpha y) \\
&= \lambda(f(x\alpha y)) \\
&\geq \lambda(f(y)) \\
&\geq (\lambda \circ f)(y) \\
&\geq \rho(y).
\end{align*}
$

This completes the proof.

Hence $\rho$ is a $T$-fuzzy $TL$-ideal of $\Gamma$-near ring.

Theorem 3.7. Let $M$ and $N$ be $\Gamma$-near rings. If $L$-subset of $A_1$ and $A_2 \in L^R$ be $T$-fuzzy $TL$-ideal of $\Gamma$-near rings of $M$ and $N$ respectively, then $A = A_1XA_2$ is a $T$-fuzzy $TL$-ideal of the direct product of $MXN$.

Proof. Let $L$-subset of $A_1$ and $A_2 \in L^R$ be $T$-fuzzy $TL$-ideal of $\Gamma$-near rings of $M$ and $N$ respectively. Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in MXN$ and $\alpha \in \Gamma$. Then,

(i) $A(0) = A_1XA_2(0) = 1.$

(ii) $
\begin{align*}
A(-(x_1, x_2)) &= A_1XA_2(-(x_1, x_2)) \\
&= A_1(-x_1, x_2)TA_2(-x_1, x_2) \\
&\geq A_1(x_1, x_2)TA_2(x_1, x_2) \\
&\geq A_1(x_1, x_2)^2 \\
&\geq A_1(x_1, x_2).
\end{align*}
$

(iii) $
\begin{align*}
A((x_1, x_2) - (y_1, y_2)) &= A_1XA_2(x_1 - y_1, x_2 - y_2) \\
&= A_1(x_1 - y_1)TA_2(x_2 - y_2) \\
&\geq A_1(x_1)TA_1(y_1)TA_2(x_2)TA_2(y_2) \\
&\geq (A_1XA_2)(x_1, x_2)TA_2(y_1, y_2) \\
&\geq A_1(x_1, x_2)TA(y_1, y_2).
\end{align*}
$
Theorem 3.8. If near ring $R$ Then, $\phi_{\mu}$ is a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring $R$, then $\phi_{\mu}$ is a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring.

Proof. Let $L$-subset $\mu \in L^R$ be a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring $R$. Suppose that, $\mu(x-y) = 0$. Then $\mu(x+y) = \mu(x) = \mu(y)$. Let $x, y, z, \mu \in L^R$ and $\alpha \in \Gamma$. Then,

(i) $\phi_{\mu}(0+\mu) = \mu(0) = 1.$

(ii) $\phi_{\mu}((x+y)+\mu) = \mu((x+y)+\mu) = \mu((x+y)+\mu) = 1.$

Hence $A = A_1X_2\alpha$ is a $T$-fuzzy $TL$-ideal of $R$. 

Notation ([11]) Let $L$-subset $\mu \in L^R$ be a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring $R$. We define $\phi_{\mu} = \frac{R}{\mu} \to [0, 1]$ by $\phi_{\mu}(x+y) = \mu(x)$ for all $x \in R$.

Theorem 3.8. If $L$-subset $\mu \in L^R$ is a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring $R$, then $\phi_{\mu}$ is a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring.

Proof. Let $L$-subset $\mu \in L^R$ be a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring $R$ and $x, y \in R$. Suppose that, $\mu(x-y) = 0$. Then $\mu(x+y) = \mu(x) = \mu(y)$. Let $x, y, z, \mu \in L^R$ and $\alpha \in \Gamma$. Then,

(i) $\phi_{\mu}(0+\mu) = \mu(0) = 1.$

(ii) $\phi_{\mu}((x+y)+\mu) = \mu((x+y)+\mu) = \mu((x+y)+\mu) = 1.$

Hence $\phi_{\mu}$ is a $T$-fuzzy $TL$-ideal of a $\Gamma$-near ring $\frac{R}{\mu}$. 

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