Operation approaches on decompositions of $\gamma$–continuous function

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Abstract
In this paper, we introduce the notions of $\alpha^* - \gamma$–set, $t - \gamma$–set, $s - \gamma$–set, $\beta^* - \gamma$–set, $C_\gamma$–continuity, $S_\gamma$–continuity and $\beta_\gamma$–continuity. Thus we have decompositions of $\gamma$–continuity.

Keywords
$\alpha - \gamma$–open, semi $- \gamma$–open, pre $- \gamma$–open, $\beta - \gamma$–open.

AMS Subject Classification
54A05, 54C08.

1. Introduction
In [13], Kasahara unified several known characterizations of compactness, nearly compact spaces and $H$-closed spaces by introducing a certain operation on a topology. By using operation Jankovic [14] investigated functions with closed graphs. Ogata [7] defined the concept of $\gamma$–open sets with an operation $\gamma$ in the manner of Kasahara [13] and introduced some new separation axioms of topological spaces. In [11], the authors introduced and investigated the notions of $\alpha - \gamma$–open sets. In [5, 6] the authors introduced and investigated the notions of semi $- \gamma$–open set, pre $- \gamma$–open set and $\beta - \gamma$–open set. A decomposition of $\gamma$–continuity is a pair of properties of functions between topological spaces with an operation $\gamma$ each of which is weaker than $\gamma$–continuity, and which are together equivalent to $\gamma$–continuity. One member of the pair is a $\gamma$–continuity dual of the other. In this paper, we introduce the notions of $\alpha^* - \gamma$–set, $t - \gamma$–set, $s - \gamma$–set, $\beta^* - \gamma$–set, $C_\gamma$–continuity, $B_\gamma$–continuity, $S_\gamma$–continuity, $\beta_\gamma$–continuity. Thus we have decompositions of $\gamma$–continuity.

2. Preliminaries
Let $(X, \tau)$ be a topological space. Let $\gamma$ be an operation on $\tau$, that is, $\gamma$ is a function from $\tau$ into the power set $\sigma(X)$ of $X$ such that $V \subset \gamma(V)$ for any $V \in \tau$ where $\gamma(V)$ denotes the value of $\gamma$ at $V$. This operation denoted by $\gamma: \tau \rightarrow \sigma(X)$. Let us take a topological space $(X, \tau)$ and $W \subset X$ with an operation $\gamma$ on $\tau$. Then $W$ is called $\gamma$–open [7] if for each $x \in W$, there exists an open neighbourhood $U$ of $x$ such that $\gamma(U) \subset W$. The empty set $\phi$ is $\gamma$–open for any operation $\gamma: \tau \rightarrow \sigma(X)$. Let $\tau_\gamma$ be the collections of all $\gamma$–open sets with $\tau_\gamma$. For any topological space $(X, \tau)$, $\tau_\gamma \subset \tau$ [7]. Complements of $\gamma$–open sets are defined as $\gamma$–closed. The $\gamma$–closure of $W \subset X$ with an operation $\gamma$ is denoted by $Cl_\gamma(W)$, is defined as

$$Cl_\gamma(W) = \cap \{B : B \text{ is } \gamma \text{–closed and } W \subset B\}.$$ 

The $\gamma$–interior of $W \subset X$ with an operation $\gamma$ on $\tau$ is denoted by $Int_\gamma(W)$, is defined as

$$Int_\gamma(W) = \cup \{B : B \text{ is a } \gamma \text{–open set and } B \subset W\}.$$ 

A topological space $X$ with an operation $\gamma$ on $\tau$ is said to be $\gamma$–regular if for each $x \in X$ and each neighbourhood $V$ of $x$, there exists an open neighbourhood $U$ of $x$ with $\gamma(U) \subset V$. According to this notion, $\tau = \tau_\gamma \iff X$ is a $\gamma$–regular space [7].

In this paper, $(X, \tau)$ and $(Y, \sigma)$ denotes topological space. Furthermore, there is no separation axioms on them unless

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3. Let $(X, \tau)$ be a topological space and $W \subseteq X$ with an operation $\gamma$ on $\tau$. Then
1. $W$ is called an $\alpha - \gamma$ open set [12] if $W \subseteq \operatorname{Int}(\operatorname{Cl}(\operatorname{Int}(W)))$,
2. $W$ is called a $\alpha - \gamma$ open set [2] if $W \subseteq \operatorname{Int}(\operatorname{Cl}(W))$,
3. $W$ is called a semi-$\gamma$ open set [10] if $W \subseteq \operatorname{Cl}(\operatorname{Int}(W))$,
4. $W$ is called a $\beta - \gamma$ open set [9] if $W \subseteq \operatorname{Cl}(\operatorname{Int}(\operatorname{Cl}(W)))$,
5. $W$ is called an $\alpha^* - \gamma$ set [4] if $\operatorname{Int}(\operatorname{Cl}(\operatorname{Int}(W))) = \operatorname{Int}(W)$,
6. $W$ is called a $C - \gamma$ set [4] if $W = U \cap V$, where $U \subseteq \tau$ and $V$ is an $\alpha^*$ set,
7. $W$ is called a $t - \gamma$ set [8] if $\operatorname{Int}(\operatorname{Cl}(W)) = \operatorname{Int}(W)$,
8. $W$ is called a $B - \gamma$ set [8] if $W = U \cap V$, where $U \subseteq \tau$ and $V$ is a $t - \gamma$ set,

Definition 2.2. Let $(X, \tau)$ be a topological space and $W \subseteq X$ with an operation $\gamma$ on $\tau$. Then
1. $W$ is called an $\alpha - \gamma$ open set [11] if $W \subseteq \operatorname{Int}(\operatorname{Cl}(\operatorname{Int}(\operatorname{Cl}(W))))$,
2. $W$ is called a $\alpha - \gamma$ open set [6] if $W \subseteq \operatorname{Int}(\operatorname{Cl}(\operatorname{Cl}(W)))$,
3. $W$ is called a semi-$\gamma$ open set [5] if $W \subseteq \operatorname{Cl}(\operatorname{Int}(\operatorname{Cl}(W)))$,
4. $W$ is called a $\beta - \gamma$ open set [6] if $W \subseteq \operatorname{Cl}(\operatorname{Cl}(\operatorname{Int}(\operatorname{Cl}(W))))$,
5. $W$ is called a $\beta^* - \gamma$ set [3] if $\operatorname{Bet}(\gamma(W)) = W \cap \operatorname{Cl}(\operatorname{Cl}(\operatorname{Cl}(W))))$.

Definition 2.3. Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and let $\gamma : \tau \rightarrow \varphi(X)$ be the operation on $\tau$. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $\gamma$ continuous [3] (resp. $\alpha - \gamma$ continuous [11], pre-$\gamma$ continuous [6], semi-$\gamma$ continuous [5], $\beta - \gamma$ continuous [6]) if for each $x \in X$ and each open set $V$ of $Y$ containing $\gamma(x)$, there exists a $\gamma$ open set $U$ containing $x$ (resp. $\alpha - \gamma$ open set, pre-$\gamma$ open set, semi-$\gamma$ open set, $\beta - \gamma$ open set) such that $\gamma(U) \subseteq V$.

3. $C - \gamma$ sets, $B - \gamma$ sets, $S - \gamma$ sets and $\beta - \gamma$ sets

Proposition 3.3. Let $W$ be a subset of a space $(X, \tau)$ with an operator $\gamma$.
1. A semi-$\gamma$ open set $W$ is a $t - \gamma$ set if and only if $W$ is an $\alpha^* - \gamma$ set.
2. $W$ is an $\alpha - \gamma$ open set and $W$ is an $\alpha^* - \gamma$ set if and only if $W$ is $\gamma$ regular and $\alpha - \gamma$ open set.

Proof. 1. Let $W$ be a semi-$\gamma$ open and $W$ be an $\alpha^* - \gamma$ set. Since $W$ is a semi-$\gamma$ open, $\operatorname{Cl}(\operatorname{Int}(W)) = \operatorname{Cl}(W)$ and $\operatorname{Int}(\operatorname{Cl}(\operatorname{Int}(W))) = \operatorname{Int}(\operatorname{Cl}(\operatorname{Cl}(\operatorname{Int}(W)))) = \operatorname{Int}(W)$. Therefore, $W$ is a $t - \gamma$ set.
2. Let $W$ be an $\alpha - \gamma$ open set and $W$ be an $\alpha^* - \gamma$ set. By Proposition 1 and the definition of $\alpha - \gamma$ open set, we have $\operatorname{Int}(\operatorname{Cl}(\operatorname{Int}(W))) = W$ and hence $\operatorname{Int}(\operatorname{Cl}(\operatorname{Int}(W))) = \operatorname{Int}(\operatorname{Cl}(\operatorname{Cl}(\operatorname{Int}(W)))) = W$.

The converse is obvious.

Definition 3.4. Let $(X, \tau)$ be a topological space and $W \subseteq X$ with an operation $\gamma$ on $\tau$. Then
1. $W$ is called a $C - \gamma$ set if $W = U \cap V$, where $U \subseteq \tau$ and $V$ is an $\alpha^*$ set,
2. $W$ is called a $B - \gamma$ set if $W = U \cap V$, where $U \subseteq \tau$ and $V$ is a $t - \gamma$ set,
3. $W$ is called a $S - \gamma$ set if $W = U \cap V$, where $U \subseteq \tau$ and $V$ is a $s - \gamma$ set,
4. $W$ is called a $\beta - \gamma$ set if $W = U \cap V$, where $U \subseteq \tau$ and $V$ is a $\beta^* - \gamma$ set,
5. $W$ is called a $\gamma - \beta - \gamma$ open set [3] if $\beta \operatorname{Int}(\gamma(W)) = \operatorname{Int}(\gamma(W))$.

Proposition 3.5. Let $(X, \tau)$ be a topological space with an operation $\gamma$ and $W \subseteq X$. Then the following hold:
1. If $W$ is a $t - \gamma$ set, then $W$ is an $\alpha^* - \gamma$ set,
2. If $W$ is a $s - \gamma$ set, then $W$ is an $\alpha^* - \gamma$ set,
3. If $W$ is a $B^* - \gamma$ set, then $W$ is both $t - \gamma$ set and $s - \gamma$ set,
4. $t - \gamma$ set and $s - \gamma$ set are independent.

Proof. Straightforward from the definitions of $\gamma$ interior and $\gamma$ closure.

Remark 3.6. The converses are false. See the following examples.

Example 3.7. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$. We define an operator $\gamma : \tau \rightarrow \varphi(X)$ by $\gamma(W) = W \cup \{a\}$ if $W \neq \emptyset$ and $\gamma(W) = W$ if $W = \emptyset$. Then $\tau_\gamma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. If we take $W = \{a\}$, then $W$ is an $\alpha^* - \gamma$ set and a $t - \gamma$ set, but it is not a $s - \gamma$ set and not a $B^* - \gamma$ set.

Example 3.8. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}\}$. We define an operator $\gamma : \tau \rightarrow \varphi(X)$ by $\gamma(W) = W$ if $W \neq \emptyset$ or $\emptyset$ if $W = \emptyset$. Then $\tau_\gamma = \{\emptyset, X\}$. If we take $W = \{a\}$, then $W$ is an $\alpha^* - \gamma$ set and a $s - \gamma$ set, but it is not a $t - \gamma$ set and not a $B^* - \gamma$ set.
**Proposition 3.9.** Let \((X, \tau)\) be a topological space with an operation \(\gamma\) and \(W \subset X\). Then the following hold:

1. If \(W\) is an \(\alpha^* - \gamma\) set, then \(W\) is a \(C_\gamma\) set.
2. If \(W\) is a \(t - \gamma\) set, then \(W\) is a \(B_\gamma\) set.
3. If \(W\) is an \(s - \gamma\) set, then \(W\) is a \(S_\gamma\) set.
4. If \(W\) is an \(\beta^* - \gamma\) set, then \(W\) is a \(\beta_\gamma\) set.

**Proof.** Let \(W\) be an \(\alpha^* - \gamma\) set. If we take \(U = X \in \tau_\gamma\), then \(W = U \cap W\) and hence \(W\) is a \(C_\gamma\) set.

Remark 3.10. The converses are false. See the following examples.

**Example 3.11.** In Example 1, if we take \(W = \{a, c\}\), then \(W\) is a \(C_\gamma\) set (resp. \(B_\gamma\) set, \(S_\gamma\) set, \(\beta_\gamma\) set), but it is not an \(\alpha^* - \gamma\) set (resp. \(t - \gamma\) set, \(s - \gamma\) set, \(\beta^* - \gamma\) set).

**Proposition 3.12.**

1. A \(B_\gamma\) set is a \(C_\gamma\) set.
2. A \(S_\gamma\) set is a \(C_\gamma\) set.
3. A \(\beta_\gamma\) set is both a \(B_\gamma\) set and a \(S_\gamma\) set.

**Remark 3.13.** The converses are false. \(B_\gamma\) set and \(S_\gamma\) set are independent notions. See the following examples.

**Example 3.14.** In Example 1, if we take \(W = \{a, b\}\), then \(W\) is a \(B_\gamma\) set, but it is not a \(S_\gamma\) set and not a \(\beta_\gamma\) set.

In Example 2, if we take \(W = \{b\}\), then \(W\) is a \(C_\gamma\) set and a \(S_\gamma\) set, but it is not a \(B_\gamma\) set and not a \(\beta_\gamma\) set.

**Proposition 3.15.** Let \((X, \tau)\) be a topological space with an operation \(\gamma\) and \(W \subset X\). Then \(\gamma - \gamma - \beta - \gamma\) set [3] and \(\beta_\gamma - \gamma\) set are equivalent.

**Proof.** Let \(W\) be a \(\beta^* - \gamma\) set. Then \(\text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(W))) = \text{Int}_\gamma(W)\). Hence by Proposition 4(4), \(W\) is a \(\beta_\gamma\) set. Therefore, \(\beta\text{Int}_\gamma(W) = W \cap \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(W))) = W \cap \text{Int}_\gamma(W) = \text{Int}_\gamma(W)\). Thus \(W\) is a \(\gamma - \gamma - \beta - \gamma\) open set.

Conversely, let \(W\) be a \(\gamma - \gamma - \beta - \gamma\) open set. Then \(\beta\text{Int}_\gamma(W) = \text{Int}_\gamma(W)\). Hence \(\beta\text{Int}_\gamma(W)\) is a \(\gamma - \gamma\) open set. Since \(W = W \cap X\), \(W\) is a \(\beta_\gamma\) set.

Remark 3.16. We have the following diagram according to sets defined above. It is shown in Examples 1-2 that the notion of \(S_\gamma\) sets is different from that of \(B_\gamma\) sets.

![Diagram](https://via.placeholder.com/150)

Theorem 3.17. For a subset \(W\) of a space \((X, \tau)\) with an operation \(\gamma\), the following properties are equivalent:

1. \(W\) is \(\gamma\) open.
2. \(W\) is an \(\alpha - \gamma\) open set and a \(C_\gamma\) set.
3. \(W\) is a \(\pre - \gamma\) open set and a \(B_\gamma\) set.
4. \(W\) is a semi \(\gamma\) open set and a \(S_\gamma\) set.
5. \(W\) is a \(\beta - \gamma\) open set and a \(\beta_\gamma\) set.

**Proof.** The proof of (1)\(\Rightarrow\)(2), (1)\(\Leftrightarrow\)(3), (1)\(\Rightarrow\)(4), (1)\(\Rightarrow\)(5) are obvious.

(5)\(\Rightarrow\)(1) Let \(W\) be a \(\beta - \gamma\) open set and a \(\beta_\gamma\) set. Since \(W\) is a \(\beta_\gamma\) set, we have \(W = U \cap V\), where \(U\) is a \(\gamma\) open set and \(V\) is a \(\beta^* - \gamma\) set. By the hypothesis, \(W\) is also \(\beta - \gamma\) open and we have

\[
W \cap \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(W))) = \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(U \cap V)))
\]

\[
\subseteq \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(U) \cap \text{Cl}_\gamma(V)))
\]

\[
= \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(U)) \cap \text{Int}_\gamma(\text{Cl}_\gamma(V)))
\]

\[
\subset \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(U))) \cap \text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(V))).
\]

Hence

\[
W = U \cap V = (U \cap V) \cap U
\]

\[
\subseteq (\text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(U)))) \cap \text{Int}_\gamma(\text{Cl}_\gamma(V)) \cap U
\]

\[
= (\text{Cl}_\gamma(\text{Int}_\gamma(\text{Cl}_\gamma(U)))) \cap U \cap \text{Int}_\gamma(\text{Cl}_\gamma(V)).
\]

Notice \(W = U \cap \text{Int}_\gamma(\text{Cl}_\gamma(V))\). Therefore, we obtain \(W = U \cap \text{Int}_\gamma(\text{Cl}_\gamma(V))\).

(2)\(\Rightarrow\)(1), (3)\(\Rightarrow\)(1), (4)\(\Rightarrow\)(1) are shown similarly.

Remark 3.18. If \((X, \tau)\) is a \(\gamma\) regular space, then the concept of \(\alpha - \gamma\) open and \(\alpha - \gamma\) open (resp. \(\pre - \gamma\) open and \(\beta - \gamma\) open, \(\beta - \gamma\) open and \(\beta - \gamma\) open, \(B_\gamma - \gamma\) set and \(B - \gamma\) set, \(C_\gamma - \gamma\) set and \(C - \gamma\) set) coincide.

**4. Decompositions of \(\gamma\) - continuity**

**Definition 4.1.** Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be a function and let \(\gamma : \tau \rightarrow \mathcal{B}(X)\) be a function on the \(\tau\). If for each \(V \in \sigma\), \(f^{-1}(V)\) is a \(C_\gamma\) set (resp. \(B_\gamma\) set, \(S_\gamma\) set, \(\beta_\gamma\) set), then \(f\) is said to be \(C_\gamma\) continuous (resp. \(B_\gamma\) continuous, \(S_\gamma\) continuous, \(\beta_\gamma\) continuous).

By Proposition 5, we get the following proposition.

**Proposition 4.2.**

1. A \(B_\gamma\) continuous function is \(C_\gamma\) continuous.
2. A \(S_\gamma\) continuous function is \(C_\gamma\) continuous,
3. A \(\beta_\gamma\) continuous is both \(B_\gamma\) continuous and \(S_\gamma\) continuous.
Theorem 4.3. For a function \( f : (X, \tau) \to (Y, \sigma) \) with the operation \( \gamma \) on \( \tau \), the following properties are equivalent:
1. \( f \) is \( \gamma \)–continuous
2. \( f \) is \( \alpha \)–\( \gamma \)–continuous and \( C_\gamma \)–continuous
3. \( f \) is pre–\( \gamma \)–continuous and \( B_\gamma \)–continuous
4. \( f \) is semi–\( \gamma \)–continuous and \( S_\gamma \)–continuous
5. \( f \) is \( \beta \)–\( \gamma \)–continuous and \( \beta_\gamma \)–continuous.

Proof. This is an immediate consequence of Theorem 1. \( \square \)

Remark 4.4. \( \alpha \)–\( \gamma \)–continuity and \( C_\gamma \)–continuity, pre–\( \gamma \)–continuity and \( B_\gamma \)–continuity, semi–\( \gamma \)–continuity and \( S_\gamma \)–continuity, \( \beta \)–\( \gamma \)–continuity and \( \beta_\gamma \)–continuity are independent of each other. See the following examples.

Example 4.5. Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, X, \{a\}, \{a, b\}\} \) and \( \sigma = \{\phi, Y, \{a\}, \{a, b\}\} \). We define an operator \( \gamma : \tau \to \mathcal{P}(X) \) by \( \gamma(W) = W \cup \{a\} \) if \( W \neq \{a\} \) and \( \gamma(W) = W \) if \( W = \{a\} \). Then \( \tau_\gamma = \{\phi, X, \{a\}, \{a, b\}\} \). Define a function \( f : (X, \tau) \to (Y, \sigma) \) as \( f(a) = f(b) = a, f(c) = c \). Then \( f \) is \( C_\gamma \)–continuous (resp. \( B_\gamma \)–continuous, semi–\( \gamma \)–continuous and \( \beta \)–\( \gamma \)–continuous), but it is not \( \alpha \)–\( \gamma \)–continuous (resp. pre–\( \gamma \)–continuous, \( S_\gamma \)–continuous and \( \beta_\gamma \)–continuous).

Example 4.6. Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, X, \{a\}, \{a, b\}\} \) and \( \sigma = \{\phi, Y, \{a\}, \{a, b\}\} \). We define an operator \( \gamma : \tau \to \mathcal{P}(X) \) by \( \gamma(W) = \text{Cl}(W) \) if \( W \neq \{a\} \) and \( \gamma(W) = \text{Int}(\text{Cl}(W)) \) if \( W = \{a\} \). Then \( \tau_\gamma = \{\phi, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\} \). Define a function \( f : (X, \tau) \to (Y, \sigma) \) as \( f(a) = f(c) = a, f(b) = b \). Then \( f \) is both \( S_\gamma \)–continuous and pre–\( \gamma \)–continuous, but it is neither semi–\( \gamma \)–continuous nor \( B_\gamma \)–continuous.

Example 4.7. Let \( X = \{a, b, c, d\} \) and \( \tau = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}, \{a, d\}\} \) and \( \sigma = \{\phi, Y, \{a\}, \{a, b\}, \{a, d\}\} \). We define an operator \( \gamma : \tau \to \mathcal{P}(X) \) by \( \gamma(W) = \text{Cl}(W) \) if \( W \neq \{a\} \) and \( \gamma(W) = \text{Int}(\text{Cl}(W)) \) if \( W = \{a\} \). Then \( \tau_\gamma = \{\phi, \{a\}, \{a, c\}, \{a, b, d\}, X\} \). Define a function \( f : (X, \tau) \to (Y, \sigma) \) as \( f(a) = f(c) = a, f(b) = f(d) = b \). Then \( f \) is \( \beta_\gamma \)–continuous, but it is not \( \beta \)–\( \gamma \)–continuous.

Example 4.8. Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, X, \{a\}, \{a, c\}, \{b, c\}\} \) and \( \sigma = \{\phi, Y, \{a\}\} \). We define an operator \( \gamma : \tau \to \mathcal{P}(X) \) by \( \gamma(W) = \text{Int}(\text{Cl}(W)) \) if \( W \neq \{a\} \) and \( \gamma(W) = X \) if \( W = \{a\} \). Then \( \tau_\gamma = \{\phi, \{a\}, X\} \). Define a function \( f : (X, \tau) \to (Y, \sigma) \) as \( f(a) = f(c) = a, f(b) = b \). Then \( f \) is \( \alpha \)–\( \gamma \)–continuous, and it is not \( C_\gamma \)–continuous.

Corollary 4.9. Let \( (X, \tau) \) be a \( \gamma \)–regular space. For a function \( f : (X, \tau) \to (Y, \sigma) \), the following properties are equivalent:
1. \( f \) is continuous
2. \( f \) is pre–continuous and \( B \)–continuous
3. \( f \) is \( \alpha \)–continuous and \( C \)–continuous

Proof. In \( \gamma \)–regular space, we have \( \tau = \tau_\gamma \). \( \square \)

5. Conclusion
A decomposition of \( \gamma \)–continuity is a pair of properties of functions between topological spaces with an operation \( \gamma \) each of which is weaker than \( \gamma \)–continuity, and which are together equivalent to \( \gamma \)–continuity. One member of the pair is a \( \gamma \)–continuity dual of the other. In this paper, we have obtain decompositions of \( \gamma \)–continuity.

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