Malaya J. Mat. 2(3)(2014) 188–196

Malaya Journal of Matematik

MJM
an international journal of mathematical sciences with computer applications...

www.malayajournal.org

ISSN : 2319-3786

Product cordial labeling for alternate snake graphs

S. K. Vaidya[a,*] and N B Vyas[b]

[a]Department of Mathematics, Saurashtra University, Rajkot-360 005, Gujarat, India.

[b]Department of Mathematics, Atmiya Institute of Technology and Science, Rajkot-360 005, Gujarat, India.

Abstract

The product cordial labeling is a variant of cordial labeling. Here we investigate product cordial labelings for alternate triangular snake and alternate quadrilateral snake graphs.

Keywords: Cordial labeling, Product cordial labeling, Snake graph.

2010 MSC: 05C78.

1 Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$. For standard terminology and notations we follow West[1].

If the vertices are assigned values subject to certain condition(s) then it is known as *graph labeling*. A mapping $f : V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the *label* of vertex v of G under f.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$and $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

A binary vertex labeling of graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if admits a cordial labeling.

The concept of cordial labeling was introduced by Cahit[2] and in the same paper he investigated several results on this newly introduced concept. A latest survey on various graph labeling problems can be found in Gallian [3].

Motivated through the concept of cordial labeling, Sundaram *et al.* [4] have introduced a labeling which has the flavour of cordial lableing but absolute difference of vertex labels is replaced by product of vertex labels.

A binary vertex labeling of graph G with induced edge labeling $f^* : E(G) \to \{0,1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a *product cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *product cordial* if it admits a product cordial labeling.

Many researchers have explored this concept, Sundaram *et al.* [4] have proved that trees, unicyclic graphs of odd order, triangular snakes, dragons, helms and union of two path graphs are product cordial. They

*Corresponding author.
E-mail address: samirkvaidya@yahoo.co.in (S. K. Vaidya), niravbvyas@gmail.com(N B Vyas).

have also proved that a graph with p vertices and q edges with $p \geq 4$ is product cordial then $q < \frac{p^2-1}{4} + 1$. Vaidya and Dani[5] have proved that the graphs obtained by joining apex vertices of k copies of stars, shells and wheels to a new vertex are product cordial while Vaidya and Kanani[6] have reported the product cordial labeling for some cycle related graphs and investigated product cordial labeling for the shadow graph of cycle C_n. The same authors have investigated some new product cordial graphs in [7]. Vaidya and Vyas [8] have investigated product cordial labeling in the context of tensor product of some standard graphs. The product cordial labelings for closed helm, web graph, flower graph, double triangular snake and gear graph are investigated by Vaidya and Barasara [9].

An alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i. That is every alternate edge of a path is replaced by C_3. *An alternate quadrilateral snake* $A(QS_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} (alternately) to new vertices v_i, w_i respectively and then joining v_i and w_i. That is every alternate edge of a path is replaced by a cycle C_4. A *double alternate triangular snake* $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, double alternate triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to two new vertices v_i and w_i. A *double alternate quadrilateral snake* $DA(QS_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, it is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to new vertices v_i, x_i and w_i, y_i respectively and adding the edges $v_i w_i$ and $x_i y_i$.

2 Main results

Theorem 2.1. *$A(T_n)$ is product cordial where $n \not\equiv 3(mod\,4)$.*

Proof. Let $A(T_n)$ be alternate triangular snake obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to new vertex v_i where $1 \leq i \leq n-1$ for even n and $1 \leq i \leq n-2$ for odd n. Therefore $V(A(T_n)) = \{u_i, v_j / 1 \leq i \leq n, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$. We note that

$$|V(A(T_n))| = \begin{cases} \frac{3n}{2} & , n \equiv 0(\text{mod } 2); \\ \frac{3n-1}{2} & , n \equiv 1(\text{mod } 2). \end{cases},$$

$$|E(A(T_n))| = \begin{cases} 2n-1 & , n \equiv 0(\text{mod } 2); \\ 2n-2 & , n \equiv 1(\text{mod } 2). \end{cases}$$

We define $f : V(A(T_n)) \to \{0,1\}$ as follows.

Case 1: $n \equiv 0(mod\,4)$

<u>For $1 \leq i \leq n$:</u>

$$f(u_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{2}; \\ 1 & , otherwise. \end{cases}$$

<u>For $1 \leq i \leq \frac{n}{2}$:</u>

$$f(v_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) = v_f(1) = \frac{3n}{4}\,, e_f(0) = e_f(1) + 1 = n$$

Case 2: $n \equiv 1(mod\,4)$

<u>For $1 \leq i \leq n$:</u>

$$f(u_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{2}; \\ 1 & , otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n-1}{2}$:

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-1}{4}; \\ 1 & ,otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = \frac{3n+1}{4}, e_f(0) = e_f(1) = n - 1$$

Case 3: $n \equiv 2(mod\,4)$

For $1 \leq i \leq n$:

$$f(u_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-2}{2}; \\ 1 & ,otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n-2}{2}$:

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-2}{4}; \\ 1 & ,otherwise. \end{cases}$$

For $i = \dfrac{n}{2}$

$$f(v_i) = 0$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = \frac{3n+2}{4}, e_f(0) = e_f(1) + 1 = n$$

Thus, in each case we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $A(T_n)$ is a product cordial graph where $n \not\equiv 3(mod\,4)$. □

Remark 2.1. *$A(T_n)$ is not product cordial graph for $n \equiv 3(mod\,4)$. Because to satisfy the vertex condition for product cordial labeling it is essential to assign label 0 to $\frac{3n-1}{4}$ vertices out of $\frac{3n-1}{2}$ vertices. The vertices with label 0 will give rise to at least $n+2$ edges with label 0 and n edges with label 1. Consequently $|e_f(0) - e_f(1)| \geq 2$.*

Example 2.1. *$A(T_{10})$ and its product cordial labeling is shown in below Figure 1.*

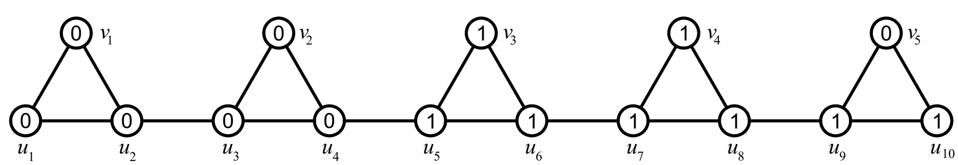

Figure 1

Theorem 2.2. *$A(QS_n)$ is product cordial where $n \not\equiv 2(mod\,4)$.*

Proof. Let $A(QS_n)$ be an alternate quadrilateral snake obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} (alternately) to new vertices v_i, w_i respectively and then joining v_i and w_i where $1 \leq i \leq n-1$ for even n and $1 \leq i \leq n-2$ for odd n. Therefore $V(A(T_n)) = \{u_i, v_j, w_j / 1 \leq i \leq n, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$. We note that

$$|V(A(QS_n))| = \begin{cases} 2n & ,n \equiv 0(\text{mod } 2); \\ 2n-1 & ,n \equiv 1(\text{mod } 2). \end{cases},$$

$$|E(A(QS_n))| = \begin{cases} \frac{5n-2}{2} & ,n \equiv 0(\text{mod } 2); \\ \frac{5n-5}{2} & ,n \equiv 1(\text{mod } 2). \end{cases}$$

We define $f : V(A(QS_n)) \rightarrow \{0, 1\}$ as follows.

Case 1: $n \equiv 0(mod\,4)$

For $1 \leq i \leq n$:

$$f(u_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n}{2}; \\ 1 & ,otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n}{2}$:

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n}{4}; \\ 1 & ,otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) = v_f(1) = n\,,\, e_f(0) = e_f(1) + 1 = \tfrac{5n}{4}$$

Case 2: $n \equiv 1(mod\,4)$

For $1 \leq i \leq n$:

$$f(u_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-1}{2}; \\ 1 & ,otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n-1}{2}$:

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-1}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-1}{4}; \\ 1 & ,otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = n\,,\, e_f(0) = e_f(1) = \frac{5n-5}{4}$$

Case 3: $n \equiv 3(mod\,4)$

For $1 \leq i \leq n$:

$$f(u_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{2}; \\ 1 & ,otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n-3}{2}$:

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

For $i = \dfrac{n-1}{2}$

$$f(v_i) = 0, f(w_i) = 0$$

In view of above defined labeling patterns we have

$$v_f(0)+1 = v_f(1) = n\ ,\ e_f(0) = e_f(1)+1 = \frac{5n-3}{2}$$

Thus in each case we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $A(QS_n)$ is a product cordial graph where $n \not\equiv 2(mod\, 4)$. □

Remark 2.2. *$A(QS_n)$ is not product cordial graph for $n \equiv 2(mod\, 4)$. Because to satisfy the vertex condition for product cordial labeling it is essential to assign label 0 to n vertices out of $2n$ vertices. The vertices with label 0 will give rise to at least $\frac{5n+2}{4}$ edges with label 0 and $\frac{5n-6}{4}$ edges with label 1. Consequently $|e_f(0) - e_f(1)| \geq 2$.*

Example 2.2. *$A(QS_{11})$ and its product cordial labeling. Figure 2.*

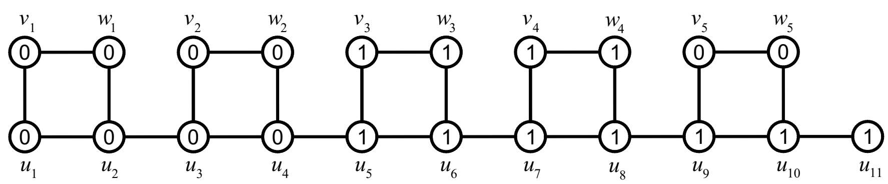

Figure 2

Theorem 2.3. *$DA(T_n)$ ia a product cordial graph where $n \not\equiv 2(mod\, 4)$.*

Proof. Let G be a double alternate triangular snake $DA(T_n)$ then $V(G) = \{u_i, v_j, w_j / 1 \leq i \leq n, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$.
We note that

$$|V(G)| = \begin{cases} 2n & , n \equiv 0(\text{mod } 2); \\ 2n-1 & , n \equiv 1(\text{mod } 2). \end{cases},$$

$$|E(G)| = \begin{cases} 3n-1 & , n \equiv 0(\text{mod } 2); \\ 3n-3 & , n \equiv 1(\text{mod } 2). \end{cases}$$

We define $f : V(A(QS_n)) \rightarrow \{0, 1\}$ as follows.
Case 1: $n \equiv 0(mod\, 4)$

<u>For $1 \leq i \leq n$:</u>

$$f(u_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{2}; \\ 1 & , otherwise. \end{cases}$$

<u>For $1 \leq i \leq \frac{n}{2}$:</u>

$$f(v_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) = v_f(1) = n\ ,\ e_f(0) = e_f(1)+1 = \frac{3n}{2}$$

Case 2: $n \equiv 1(mod\, 4)$

<u>For $1 \leq i \leq n$:</u>

$$f(u_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{2}; \\ 1 & , otherwise. \end{cases}$$

<u>For $1 \leq i \leq \frac{n-1}{2}$:</u>

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-1}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-1}{4}; \\ 1 & ,otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = n \text{ , } e_f(0) = e_f(1) = \frac{3n-3}{2}$$

Case 3: $n \equiv 3(mod\, 4)$

For $1 \leq i \leq n-1$:

$$f(u_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{2}; \\ 1 & ,otherwise. \end{cases}$$

$$f(u_n) = 0$$

For $1 \leq i \leq \dfrac{n-3}{2}$:

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

For $i = \dfrac{n-1}{2}$

$$f(v_i) = 1, f(w_i) = 0$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = n \text{ , } e_f(0) = e_f(1) = \frac{3n-3}{2}$$

Thus, in each case we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $DA(T_n)$ is a product cordial graph where $n \not\equiv 2(mod\, 4)$. □

Remark 2.3. *$DA(T_n)$ is not product cordial graph for $n \equiv 2(mod\, 4)$. Because to satisfy the vertex condition for product cordial labeling it is essential to assign label 0 to n vertices out of $2n$ vertices. The vertices with label 0 will give rise to at least $\frac{3n+2}{2}$ edges with label 0 and $\frac{3n-4}{2}$ edges with label 1. Consequently $|e_f(0) - e_f(1)| \geq 2$.*

Example 2.3. *$DA(T_{11})$ and its product cordial labeling is shown in Figure 3.*

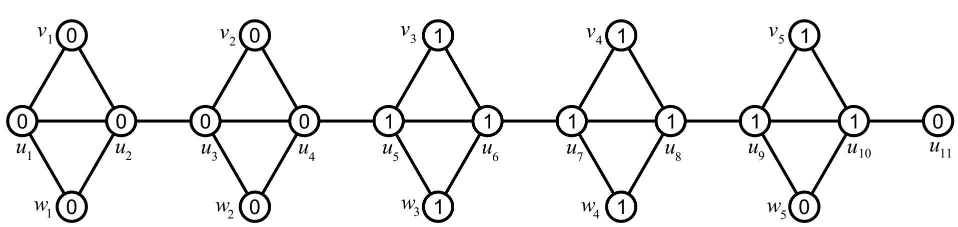

Figure 3

Theorem 2.4. *$DA(QS_n)$ is a product cordial graph where $n \not\equiv 2(mod\, 4)$.*

Proof. Let G be a double alternate quadrilateral snake $DA(T_n)$ then

$V(G) = \{u_i, v_j, w_j, x_j, y_j / 1 \leq i \leq n, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$. We note that

$|V(G)| = \begin{cases} 3n & , n \equiv 0(\text{mod } 2); \\ 3n-2 & , n \equiv 1(\text{mod } 2). \end{cases}$,

$|E(G)| = \begin{cases} 4n-1 & , n \equiv 0(\text{mod } 2); \\ 4n-4 & , n \equiv 1(\text{mod } 2). \end{cases}$

We define $f : V(G) \to \{0, 1\}$ as follows.
Case 1: $n \equiv 0(mod\, 4)$

For $1 \leq i \leq n$:

$$f(u_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{2}; \\ 1 & , otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n}{2}$:

$$f(v_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(x_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(y_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n}{4}; \\ 1 & , otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) = v_f(1) = \frac{3n}{2}, e_f(0) = e_f(1) + 1 = 2n$$

Case 2: $n \equiv 1(mod\, 4)$

For $1 \leq i \leq n$:

$$f(u_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{2}; \\ 1 & , otherwise. \end{cases}$$

For $1 \leq i \leq \dfrac{n-1}{2}$:

$$f(v_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(x_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{4}; \\ 1 & , otherwise. \end{cases}$$

$$f(y_i) = \begin{cases} 0 & , 1 \leq i \leq \frac{n-1}{4}; \\ 1 & , otherwise. \end{cases}$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = \frac{3n-1}{2}, e_f(0) = e_f(1) = 2n - 2$$

Case 3: $n \equiv 3(mod\, 4)$

For $1 \leq i \leq n - 1$:

$$f(u_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{2}; \\ 1 & ,otherwise. \end{cases}$$

$$f(u_n) = 0$$

<u>For $1 \leq i \leq \dfrac{n-3}{2}$:</u>

$$f(v_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(w_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(x_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

$$f(y_i) = \begin{cases} 0 & ,1 \leq i \leq \frac{n-3}{4}; \\ 1 & ,otherwise. \end{cases}$$

<u>For $i = \dfrac{n-1}{2}$</u>

$$f(v_i) = 1, f(w_i) = 1$$

$$f(x_i) = 0, f(y_i) = 0$$

In view of above defined labeling patterns we have

$$v_f(0) + 1 = v_f(1) = \frac{3n-1}{2}, e_f(0) = e_f(1) = 2n - 2$$

Thus, in each case we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $DA(QS_n)$ is a product cordial graph where $n \not\equiv 2(mod\, 4)$. □

Remark 2.4. *$DA(QS_n)$ is not product cordial graph for $n \equiv 2(mod\, 4)$. Because to satisfy the vertex condition for product cordial labeling it is essential to assign label 0 to $\frac{3n}{2}$ vertices out of $3n$ vertices. The vertices with label 0 will give rise to at least $2n+1$ edges with label 0 and $2n-2$ edges with label 1. Consequently $|e_f(0) - e_f(1)| \geq 2$.*

Example 2.4. *$DA(QS_8)$ and its product cordial labeling is shown in Figure 4.*

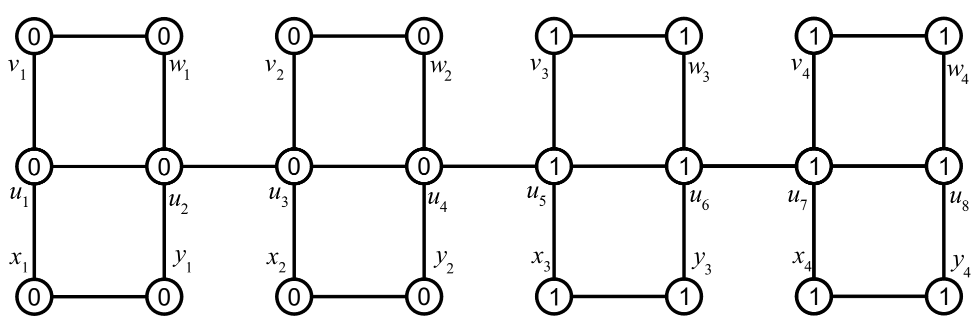

Figure 4

3 Concluding remarks

The labeling of discrete structures is one of the potential areas of research. Here we investigate product cordial labeling for some alternate snake graphs. To derive similar results for other graph families is an open area of research.

4 Acknowledgement

Our thanks are due to the anonymous referees for their constructive comments on the first draft of this paper.

References

[1] D. B. West, *Introduction to Graph Theory*, Printice-Hall of India, 2001.

[2] I. Cahit, Cordial Graphs: A weaker version of graceful and harmonious Graphs, *Ars Combinatoria*, 23(1987), 201-207.

[3] J. A. Gallian, A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, 16(#DS6), 2013.

[4] M. Sundaram, R. Ponraj and S. Somsundaram, Product cordial labeling of graphs, *Bull. Pure and Applied Sciences(Mathematics and Statistics)*, 23E(2004), 155-163.

[5] S. K. Vaidya and N. A. Dani, Some new product cordial graphs, *Journal of App. Comp. Sci. Math.*, 8(4)(2010), 63-66.

[6] S. K. Vaidya and K. K. Kanani, Some cycle related product cordial graphs, *International Journal of Algorithms*, 3(1)(2010), 109-116.

[7] S. K. Vaidya and K. K. Kanani, Some new product cordial graphs, *Mathematics Today*, 27 (2011), 64-70.

[8] S. K. Vaidya and N. B. Vyas, Product Cordial Labeling in the Context of Tensor Product of Graphs, *Journal of Mathematics Research*, 3(3)(2011), 83-88.

[9] S. K. Vaidya and C. M. Barasara, Further results on product cordial labeling, *International Journal of Math. Combin.*, 3(2012), 64-71.

Received: December 6, 2013; *Accepted*: January 11, 2014

UNIVERSITY PRESS

Website: http://www.malayajournal.org/