Operation on $\hat{\Omega}$-closed sets

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Abstract
This work is based on operation in a topological space. An operation has been extended to the class of $\hat{\Omega}$-open sets. The new class of $\gamma_{\Omega}$-open sets has been introduced and two kinds of closures such as, $\gamma_{\Omega} Cl$ and $(\hat{\Omega} Cl)_{\gamma}$ are studied. Necessary basic properties have been derived. Moreover, $\hat{\Omega}$-regular operation on $\hat{\Omega} O(X, \tau)$ has been introduced in which intersection of any two $\gamma_{\Omega}$-closed sets is $\gamma_{\Omega}$-closed. Also three types of separation axioms are defined and few results on them have been derived.

Keywords
$\gamma_{\Omega}$-open set, $\gamma_{\Omega} Cl$, $(\hat{\Omega} Cl)_{\gamma}$, $\hat{\Omega}$-regular operation, $\hat{\Omega}$-open operation, $\gamma_{\Omega} T_i$; spaces $(i = 0, 1, 2)$.

AMS Subject Classification
54A05, 54A10, 54D10.

1. Introduction

Generalized open sets play a vital role in research area of General Topology. Levine [7] introduced the concept of semi-open sets in topology. In 1987, Bhattacharyya and Lahiri [3] used semi-open sets to define the notion of semi-generalized closed sets. Kasahara [8] introduced the notion of an $\alpha$-operation approaches on a class $\tau$ of sets and studied the concept of $\alpha$-continuous functions with $\alpha$-closed graphs and $\alpha$-compact spaces. Jankovic [5] introduced the concept of $\alpha$-closure of a set in $X$ via $\alpha$-operation and investigated further characterizations of a function with $\alpha$-closed graph. Later Ogata [10] defined and studied the concept of $\gamma$-open sets and applied it to investigate operation-functions and operation-separation. Recently several researchers developed many concepts of operation $\gamma$ in a space $X$. Krishnan, Gangster and Balachandran [9] introduced and studied the concept of the operation $\gamma$ on the class of all semiopen sets of $(X, \tau)$ and defined the notion of semi $\gamma$-open sets and investigated some of their properties. An, Cuong and Maki [11] defined and investigated an operation $\gamma$ on the class of all preopen sets of $(X, \tau)$ and introduced the notion of pre-$\gamma$-open sets and developed some of their properties. Asaad [2] defined the notion of an operation $\gamma$ on the class of generalized open sets in $(X, \tau)$ and studied some of its applications. Recently, the concept of $\hat{\Omega}$-closed set was introduced and investigated by Lellis Thivagar et al. [6]. In this paper, the concept of an operation $\gamma$ has been extended to the class of $\hat{\Omega}$-open sets and it leads to the introduction of the notion of $\gamma_{\Omega}$-open sets on a topological spaces $(X, \tau)$. Furthermore, some basic properties of $\gamma_{\Omega}$-Closures have been derived. In last Section, $\gamma_{\Omega} T_i$; spaces where $i \in \{0, 1, 2\}$ are introduced and investigated using the operation $\gamma$ on $\tau_{\hat{\Omega}}$.

2. Preliminaries

In this section, some definitions and results that are used in this work have been dealt. Throughout this paper, $(X, \tau)$ or $X$ represents a topological space on which no separation axioms are assumed, unless otherwise mentioned.

Definition 2.1. [7] A subset $A$ of a topological space $(X, \tau)$ is called a semi-open set if $A \subseteq c l(\text{int}(A))$. $SO(X)$ denotes the set of all semi-open sets in $(X, \tau)$. It’s complement is known as a semi-closed set on $X$.

Definition 2.2. ([11], Definition 2.2) A subset $A$ of $X$ is called a $\delta$-closed set in a topological space $(X, \tau)$ if $A = \delta c l(A)$,
where $\delta \text{cl}(A) = \{ x \in X : \text{int}(\text{cl}(U)) \cap A \neq \emptyset, U \in O(X,x) \}$. The complement of a $\delta$-closed set in $(X,\tau)$ is denoted by $\delta X(X)$. From [4], lemma 3, $\delta \text{cl}(A) = \bigcap \{ F \in \delta X(X) : A \subseteq F \}$ and from corollary 4, $\delta \text{cl}(A)$ is a $\delta$-closed set for a subset $A$ in a topological space $(X,\tau)$.

Definition 2.3. ([6], Definition 3.1) Let $(X,\tau)$ be a topological space. $A$ is said to be $\hat{\Omega}$-closed set if $\hat{\text{cl}}(A) \subseteq U$ when $A \subseteq U$, where $U$ is a semi-open subset of $X$. The complement of $\hat{\Omega}$-closed set is $\hat{\Omega}$-open set.

Definition 2.4. ([6], Definition 5.1) Let $A$ be a subset of a topological space $(X,\tau)$. Then $\hat{\text{cl}}(A)$ is defined to be the intersection of all $\hat{\Omega}$-closed sets containing $A$ and it is denoted by $\hat{\text{cl}}(A)$. That is $\hat{\text{cl}}(A) = \bigcap \{ F : A \subseteq F \in \hat{\text{c}}(X) \}$. Always $A \subseteq \hat{\text{cl}}(A)$.

Remark 2.5. ([6], Remark 5.2) From the definition and Theorem 4.16, arbitrary intersection of $\hat{\Omega}$-closed sets in a topological space $(X,\tau)$ is $\hat{\Omega}$-closed set in $(X,\tau)$. $\hat{\text{cl}}(A)$ is the smallest $\hat{\Omega}$-closed set containing $A$.

Theorem 2.6. ([6], Theorem 5.3) Let $A$ be any subset of a topological space $(X,\tau)$. Then $A$ is a $\hat{\Omega}$-closed set in $(X,\tau)$ if and only if $A = \hat{\text{cl}}(A)$.

Theorem 2.7. ([6], Theorem 5.11) In a topological space $(X,\tau)$, for $x \in X$, $x \in \hat{\text{cl}}(A)$ if and only if $U \cap A \neq \emptyset$ for every $\hat{\Omega}$-open set $U$ containing $x$.

Definition 2.8. ([8]) Let $(X,\tau)$ be a topological space. An operation $\gamma$ on the topology $\tau$ is a mapping from $\tau \rightarrow P(X)$ such that $V \subseteq V'$ for each $V \in \tau$, where $V'$ denotes the value of $\gamma$ at $V$. It is denoted by $\gamma : \tau \rightarrow P(X)$.

Notation 2.9.

i) $U \subseteq \hat{\Omega}O(X,x)$ denotes the set of all $\hat{\Omega}$-open sets in $(X,\tau)$containing $x$.

ii) $\hat{\Omega}O(X,\tau)$ or $\hat{\Omega}O(X)$ or $\tau_{\Omega}$ denotes the set of all $\hat{\Omega}$-open sets in a topological space $(X,\tau)$.

iii) The closure (res.interior,complement)of $A$ is denoted by $\text{cl}(A)$, $\text{res.int}(A)$, $A^c$.

iv) $SO(X)$ denotes the set of all semi-open sets in a topological space $(X,\tau)$.

3. Operation on $\hat{\Omega}O(X,\tau)$

Definition 3.1. A function $\gamma : \hat{\Omega}O(X,\tau) \rightarrow P(X)$ is called an operation on $\hat{\Omega}O(X,\tau)$, if $U \subseteq \gamma(U)$ for every set $U \in \hat{\Omega}O(X,\tau)$.

Remark 3.2. For any operation $\gamma : \hat{\Omega}O(X,\tau) \rightarrow P(X)$, $\gamma(X) = X$, and $\gamma(\emptyset) = \emptyset$.

Definition 3.3. A non-empty subset $A$ of $X$ is called $\gamma_{\Omega}$-open set if for each $x \in A$, there exists an $\hat{\Omega}$-open set $U$ such that $x \in U$ and $\gamma(U) \subseteq A$. The complement of $\gamma_{\Omega}$-open set is $\gamma_{\Omega}$-closed set. Assume that the empty set $\emptyset$ is always $\gamma_{\Omega}$-open for any operation $\gamma$ on $\hat{\Omega}O(X,\tau)$. $\tau_{\Omega}$ denotes the set of all $\gamma_{\Omega}$-open sets on $(X,\tau)$. $\tau_{\emptyset} = \{ \emptyset \} \cup \{ \hat{\text{cl}}(A) \}$ for each $x \in A$ there exists an $\hat{\Omega}$-open set $U \ni x \in U$ and $\gamma(U) \subseteq A$.

Example 3.4. $X = \{ a, b, c, d \}$, $\tau = \{ \emptyset, \{ a, b, c \}, \{ a, b, c, d \}, X \}$, $\hat{\Omega}O(X) = \{ \emptyset, \{ b \}, \{ c, d \}, X \}$, $\gamma(U) = \emptyset$, $\gamma(\{ b \}) = \{ b \}$, $\gamma(\{ c \}) = \{ c \}$, $\gamma(\{ b, c \}) = \{ b, c, d \}$, $\gamma(X) = X$. Here, the operation $\gamma$ on $\tau_{\emptyset}$ is $\gamma_{\Omega}$-regular.

Theorem 3.5. Arbitrary union of $\gamma_{\Omega}$-open sets is a $\gamma_{\Omega}$-open set in a topological space $X$.

Proof. Let $\{ A_\alpha \}_{\alpha \in J}$ be any family of $\gamma_{\Omega}$-open sets in a space $(X,\tau)$. Let $A = \bigcup_{\alpha \in J} A_\alpha$ and $x \in A$ be arbitrary. Then $x \in A_\alpha$ for some $\alpha \in J$. By the definition of $\gamma_{\Omega}$-open, there exist $U \in \hat{\Omega}O(X,x)$ such that $\gamma(U) \subseteq A_\alpha \subseteq \bigcup_{\alpha \in J} A_\alpha = A$. Therefore, $A$ is $\gamma_{\Omega}$-open.

Remark 3.6. Arbitrary intersection of $\gamma_{\Omega}$-closed sets is a $\gamma_{\Omega}$-closed set in a topological space $X$.

Example 3.7. The intersection of any two $\gamma_{\Omega}$-open sets is not necessarily $\gamma_{\Omega}$-open set-in $(X,\tau)$. Let $X = \{ a, b, c \}$ and $P(X) = \hat{\Omega}O(X,\tau)$. Define an operation $\gamma : \hat{\Omega}O(X,\tau) \rightarrow P(X)$ as follows. For every $U \in \hat{\Omega}O(X,\tau)$

$$
\gamma(U) = \begin{cases} 
U & \text{for } U \neq \{ a \} \\
\{ a, b \} & \text{for } U = \{ a \}
\end{cases}
$$

Here $\{ a, b \}$ and $\{ a, c \}$ are $\gamma_{\Omega}$-open sets but $\{ a \}$ is not a $\gamma_{\Omega}$-open set.

Proposition 3.8. Every $\gamma_{\Omega}$-open set is $\hat{\Omega}$-open in a space $X$.

Proof. Let $A$ be any $\gamma_{\Omega}$-open subset of $X$. Let $x \in A$ be arbitrary. Then there exists $\hat{\Omega}$-open set $U_x$ containing $x$ such that $U_x \subseteq \gamma(U_x) \subseteq A$. Then $\bigcup_{x \in A} U_x = A$. By ([6], Theorem 4.16), $A$ is $\hat{\Omega}$-open subset of $X$.

Remark 3.9. From Example 3.7, every $\hat{\Omega}$-open is not necessarily $\gamma_{\Omega}$-open as $\{ a \} \in \hat{\Omega}O(X)$ and $\{ a \} \notin \tau_{\Omega}$. It turns out to find a space in which $\hat{\Omega}O(X) = \tau_{\Omega}$.

Definition 3.10. A space $(X,\tau)$ with an operation $\gamma$ on $\hat{\Omega}O(X,\tau)$ is called $\gamma_{\Omega}$-regular if for each $x \in X$ and for each $U \in \hat{\Omega}O(X,x)$, there exists an $\hat{\Omega}$-open set $V$ such that $x \in V$ and $\gamma(V) \subseteq U$.

Example 3.11. $X = \{ a, b, c, d \}$, $\tau = \{ \emptyset, \{ b, c \}, \{ a, b, c \}, \{ b, c, d \}, X \}$, $\hat{\Omega}O(X) = \{ \emptyset, \{ b \}, \{ c \}, \{ b, c \}, X \}$, $\gamma(U) = \emptyset$, $\gamma(\{ b \}) = \{ b \}$, $\gamma(\{ c \}) = \{ c \}$, $\gamma(\{ b, c \}) = \{ b, c, d \}$, $\gamma(X) = X$. Here, the operation $\gamma$ on $\tau_{\Omega}$ is $\gamma_{\Omega}$-regular.
Theorem 3.12. Let \((X, \tau)\) be a topological space and \(\gamma : \hat{\Omega}(X, \tau) \rightarrow P(X)\) be an operation on \(\hat{\Omega}(X, \tau)\). Then the following conditions are equivalent:

i) Every \(\hat{\Omega}\)-open set is \(\gamma\)-open set.

ii) \(X\) is an \(\gamma\)-regular space.

iii) For every \(x \in X\) and for every \(U \in \hat{\Omega}(X, x)\), there exists an \(\gamma\)-open set \(V\) of \((X, \tau)\) containing \(x\) such that \(V \subseteq U\).

Proof. i) \(\Rightarrow\) ii) Let \(x \in X\) be arbitrary and \(U \in \hat{\Omega}(X, x)\). By hypothesis, there exists \(V \in \hat{\Omega}(X, x)\) such that \(\gamma(V) \subseteq U\).

ii) \(\Rightarrow\) iii) Let \(x\) be any point of \(X\) and \(U \in \hat{\Omega}(X, x)\). By hypothesis, there exists \(\hat{\Omega}\)-open set \(V\) such that \(x \in V\) and \(\gamma(V) \subseteq U\). Again apply hypothesis to the set \(V\). Then, there exists \(\hat{\Omega}\)-open set \(V_1 \in \hat{\Omega}(X, x)\) such that \(\gamma(V_1) \subseteq V\). Then, \(V\) is \(\gamma\)-open set containing \(x\) such that \(V \subseteq U\).

iii) \(\Rightarrow\) i) Let \(U\) be any \(\hat{\Omega}\)-open set in \(X\) and \(x \in U\) be arbitrary. By hypothesis, there exists \(\gamma\)-open set \(V\) containing \(x\) such that \(V \subseteq U\). By Theorem 3.5, \(U = \bigcup_{x \in U} V_x\) is \(\gamma\)-open.

Definition 3.13. Let \((X, \tau)\) be any topological space. An operation \(\gamma\) on \(\hat{\Omega}(X, \tau)\) is called \(\hat{\Omega}\)-open if for each \(x \in X\) and for every \(U \in \hat{\Omega}(X, x)\), there exists a \(\gamma\)-open set \(V\) containing \(x\) such that \(V \subseteq \gamma(U)\).

Example 3.14. \(X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}\). \(\hat{\Omega}(X) = \{\emptyset, \{a\}, \{a, b\}, X\}\). \(\gamma\) is defined by \(\gamma(\emptyset) = \emptyset\), \(\gamma(\{a\}) = \{a\}, \gamma(\{a, b\}) = \{a, b\}, \gamma(X) = X\). Here, the operation \(\gamma\) on \(\tau_{\hat{\Omega}}\) is \(\hat{\Omega}\)-regular.

Definition 3.15. Let \((X, \tau)\) be any topological space. An operation \(\gamma\) on \(\hat{\Omega}(X, \tau)\) is called \(\hat{\Omega}\)-regular if for each \(x \in X\) and for every pair of sets \(U_1, U_2 \in \hat{\Omega}(X, x)\), there exists a set \(V \in \hat{\Omega}(X, x)\) such that \(\gamma(V) \subseteq U_1 \cap U_2\).

Example 3.16. \(X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{a, b\}, X\}\). \(\hat{\Omega}(X) = \{\emptyset, \{a\}, \{a, b\}, X\}\). \(\gamma\) is defined by \(\gamma(\emptyset) = \emptyset\), \(\gamma(\{a\}) = \{a\}, \gamma(\{a, b\}) = \{a, b\}, \gamma(X) = X\). Here, an operation \(\gamma\) on \(\tau_{\hat{\Omega}}\) is \(\hat{\Omega}\)-regular.

Proposition 3.17. Intersection of any two \(\gamma\)-open sets is a \(\gamma\)-open in a \(\hat{\Omega}\)-regular operation on \(\hat{\Omega}(X, \tau)\).

Proof. Let \(U\) and \(V\) be any two \(\gamma\)-open sets in \(X\). Let \(x \in U \cap V\) be any point. Then, \(x \in U\) and \(x \in V\). By the definition, there exists \(U_1 \in \hat{\Omega}(X, x)\) such that \(\gamma(U_1) \subseteq U\). Similarly for the set \(V\), there exists \(U_2 \in \hat{\Omega}(X, x)\) such that \(\gamma(U_2) \subseteq V\). Now \(\gamma(U_1) \cap \gamma(U_2) \subseteq U \cap V\). By hypothesis, there exists \(\hat{\Omega}\)-open set \(W\) containing \(x\) such that \(\gamma(W) \subseteq \gamma(U_1) \cap \gamma(U_2) \subseteq U \cap V\). Hence \(U \cap V\) is \(\gamma\)-open subset of \(X\).

Remark 3.18. By Proposition 3.17, the family of all \(\gamma\)-open sets satisfy the axioms topology provided an operation \(\gamma\) is a \(\hat{\Omega}\)-regular.
Proof. Let $A$ be any subset of $X$ and $F = (\hat{\Omega}C)(A)$. Always, $F \subseteq (\hat{\Omega}C)(F)$. If $x \notin F$, then there exists $U \in \hat{\Omega}O(X, x)$ such that $\gamma(U) \cap F = \emptyset$. Then, $U \cap F = \emptyset$. By Theorem 2.7, $x \notin (\hat{\Omega}C)(F)$. Therefore, $F = (\hat{\Omega}C)(F)$. By Theorem 2.6, $F$ is $\hat{\Omega}$-closed.

**Lemma 4.7.** In a topological space $(X, \tau)$ with an operation $\gamma$ on $\hat{\Omega}O(X, \tau)$, the following statements hold for any two subsets $A$ and $B$ of $X$.

i) $A \subseteq (\hat{\Omega}C)(A) \subseteq \gamma_{\hat{\Omega}}(A)$.

ii) $A$ is $\gamma_{\hat{\Omega}}$-closed if and only if $(\hat{\Omega}C)(A) = A$.

iii) If $A \subseteq B$, $(\hat{\Omega}C)(A) \subseteq (\hat{\Omega}C)(B)$.

iv) $(\hat{\Omega}C)(A \cap B) \subseteq (\hat{\Omega}C)(A) \cap (\hat{\Omega}C)(B)$.

v) $(\hat{\Omega}C)(A) \cup (\hat{\Omega}C)(B) \subseteq (\hat{\Omega}C)(A \cup B)$.

**Theorem 4.8.** If $\gamma$ is an $\hat{\Omega}$-regular operation on $\hat{\Omega}O(X, \tau)$, then for any two subsets $A, B$ of $X$ the following results hold.

i) $\gamma_{\hat{\Omega}}(A) \cup \gamma_{\hat{\Omega}}(B) = \gamma_{\hat{\Omega}}(A \cup B)$.

ii) $(\hat{\Omega}C)(A) \cup (\hat{\Omega}C)(B) = (\hat{\Omega}C)(A \cup B)$.

**Theorem 4.9.** Let $A$ be any subset of a topological space $(X, \tau)$. If $\gamma$ is an $\hat{\Omega}$-open operation on $\hat{\Omega}O(X, \tau)$, then the following statements are true.

i) $(\hat{\Omega}C)(A) = \gamma_{\hat{\Omega}}(A)$.

ii) $(\hat{\Omega}C)(\gamma(\hat{\Omega}C)(A)) = (\hat{\Omega}C)(A)$.

iii) $(\hat{\Omega}C)(A)$ is $\gamma_{\hat{\Omega}}$-closed in $X$.

**Theorem 4.10.** Let $A$ be any subset of a topological space $(X, \tau)$ and $\gamma$ be an operation on $\hat{\Omega}O(X, \tau)$. Then the following statements are equivalent.

i) $A$ is $\gamma_{\hat{\Omega}}$-open set.

ii) $(\hat{\Omega}C)(X \setminus A) = X \setminus A$.

iii) $\gamma_{\hat{\Omega}}Cl(X \setminus A) = X \setminus A$.

iv) $X \setminus A$ is $\gamma_{\hat{\Omega}}$-closed set.
Proof. Assume that \( x \in \gamma_\Omega Cl(A) \cap U \) for every \( \gamma_\Omega \)-open set \( U \) and every subset \( A \) of \( X \). Let \( V \) be any \( \gamma_\Omega \)-open subset of \( X \) containing \( x \). By Proposition 3.17, \( U \cap V \) is \( \gamma_\Omega \)-open set containing \( x \). Since, \( x \in \gamma_\Omega Cl(A), \ A \cap \{U \cap V\} = \emptyset \). That is, \( \{A \cap V\} \cup V = \emptyset \). By Theorem 4.3, \( x \in \gamma_\Omega Cl(A \cup V) \).

5. Separation axioms

Definition 5.1. A topological space \((X, \tau)\) with an operation \(\gamma\) on \(\hat{\Omega}O(X, \tau)\) is called \(\gamma_\Omega-T_0\) if for any two points \(x, y\) in \(X\) such that \(x \neq y\) there exists an \(U \in \hat{\Omega}O(X, \tau)\), such that \(x \in U\) and \(y \notin \gamma(U)\) or \(y \in U\) and \(x \notin \gamma(U)\).

Definition 5.2. A topological space \((X, \tau)\) with an operation \(\gamma\) on \(\hat{\Omega}O(X, \tau)\) is called \(\gamma_\Omega-T_1\) if for any two points \(x, y\) in \(X\) such that \(x \neq y\), there exist two \(\hat{\Omega}\)-open sets \(U\) and \(V\) containing \(x\) and \(y\) respectively such that \(y \notin \gamma(U)\) and \(x \notin \gamma(V)\).

Definition 5.3. A topological space \((X, \tau)\) with an operation \(\gamma\) on \(\hat{\Omega}O(X, \tau)\) is called \(\gamma_\Omega-T_2\) if for any two points \(x, y\) in \(X\) such that \(x \neq y\), there exist two \(\hat{\Omega}\)-open sets \(U\) and \(V\) containing \(x\) and \(y\) respectively such that \(\gamma(U) \cap \gamma(V) = \emptyset\).

Theorem 5.4. Let \(\gamma\) be an \(\hat{\Omega}\)-open operation on \(\hat{\Omega}O(X, \tau)\). Then \((X, \tau)\) is an \(\gamma_\Omega-T_0\) space iff \((\hat{\Omega}Cl)_{\gamma}(\{x\}) \neq (\hat{\Omega}Cl)_{\gamma}(\{y\})\) for every pair \(x, y\) of \(X\) with \(x \neq y\).

Proof. Let \(x, y\) be any two distinct points of an \(\gamma_\Omega-T_0\) space \((X, \tau)\). Then, there exists a \(\gamma_\Omega\)-open set \(U\) such that \(x \in U\) and \(y \notin \gamma(U)\). Since \(\gamma\) is an \(\hat{\Omega}\)-open, there exists a \(\gamma_\Omega\)-open set \(V\) such that \(x \in V\) and \(V \subseteq \gamma(U)\). Therefore, \(y \notin X \setminus (U \cap V)\). Now \(X \setminus V\) is an \(\gamma_\Omega\)-closed set in \((X, \tau)\) such that \((\hat{\Omega}Cl)_{\gamma}(\{x\}) \subseteq X \setminus V\). Thus \((\hat{\Omega}Cl)_{\gamma}(\{x\}) \neq (\hat{\Omega}Cl)_{\gamma}(\{y\})\).

Conversely, if \(x, y\) are any two distinct points of \(X\) then, \((\hat{\Omega}Cl)_{\gamma}(\{x\}) \neq (\hat{\Omega}Cl)_{\gamma}(\{y\})\). Choose \(z \in X\) such that \(z \in (\hat{\Omega}Cl)_{\gamma}(\{x\})\), and \(z \notin (\hat{\Omega}Cl)_{\gamma}(\{y\})\). If \(x \in (\hat{\Omega}Cl)_{\gamma}(\{y\})\), then \((\hat{\Omega}Cl)_{\gamma}(\{x\}) \subseteq (\hat{\Omega}Cl)_{\gamma}(\{y\})\). That is, \(z \in (\hat{\Omega}Cl)_{\gamma}(\{y\})\), which is a contradiction. So, \(x \notin (\hat{\Omega}Cl)_{\gamma}(\{y\})\). Then, there exists an \(\hat{\Omega}\)-open set \(U\) containing \(x\) such that \(\gamma(U) \cap \gamma(V) = \emptyset\).

Theorem 5.5. The space \((X, \tau)\) is \(\gamma_\Omega-T_1\) if and only if for every point \(x \in X\), \(\{x\}\) is an \(\gamma_\Omega\)-closed set.

Proof. Let \((X, \tau)\) be a \(\gamma_\Omega-T_1\) space and \(x\) be any point of \(X\). Then for any point \(y\) in \(X\) such that \(x \neq y\), there exists an \(\hat{\Omega}\)-open set \(V\) such that \(y \in V\). Thus, \(y \notin \gamma(V)\) \(\subseteq X \setminus \{x\}\). This implies that \(X \setminus \{x\} = \bigcup \{\gamma(V) : y \in X \setminus \{x\}\}\). Now \(X \setminus \{x\}\) is \(\gamma_\Omega\)-open set in \((X, \tau)\) and hence \(\{x\}\) is \(\gamma_\Omega\)-closed set in \((X, \tau)\).

Conversely, let \(x, y\) in \(X\) such that \(x \neq y\). By hypothesis, we get \(X \setminus \{y\}\) and \(X \setminus \{x\}\) are \(\gamma_\Omega\)-open sets such that \(x \in X \setminus \{y\} = U\) (say) and \(y \in X \setminus \{x\} = V\) (say). Therefore, there exist \(\hat{\Omega}\)-open sets \(U\) and \(V\) such that \(x \in U\), \(y \in V\), \(\gamma(U) \subseteq X \setminus \{y\}\), and \(\gamma(V) \subseteq X \setminus \{x\}\). So, \(y \notin \gamma(U)\) and \(x \notin \gamma(V)\). This implies that \((X, \tau)\) is \(\gamma_\Omega-T_1\).

Theorem 5.6. For any topological space \((X, \tau)\) and any operation \(\gamma\) on \(\tau_\Omega\), the following properties hold.

i) Every \(\gamma_\Omega-T_2\) space is \(\gamma-T_1\).

ii) Every \(\gamma_\Omega-T_1\) space is \(\gamma-T_0\).

Proof. It follows from definitions.

6. Conclusion

In this paper, an attempt has been made to define operation on the class of \(\hat{\Omega}\)-open sets. With the help of this operation, the new class of \(\gamma_\Omega\)-open sets has been introduced and two kinds of closures such as, \(\gamma_\Omega Cl\) and \((\hat{\Omega}Cl)_{\gamma}\) studied. Their basic properties have been derived. Moreover, it is shown by an example that intersection of any two \(\gamma_\Omega\)-closed sets is not necessarily a \(\gamma_\Omega\)-closed but that holds in a \(\hat{\Omega}\)-regular operation on \(\hat{\Omega}O(X, \tau)\) has been derived.

References


