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Nano $\mathscr{M}$ seperation axioms

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Abstract
In this paper, the basic properties of $\mathfrak{N}\mathscr{M}T_0$, $\mathfrak{N}\mathscr{M}T_1$, $\mathfrak{N}\mathscr{M}T_2$, $\mathfrak{N}\mathscr{M}reg$ and $\mathfrak{N}\mathscr{M}nor$ spaces are established. Further, the study of nano $\mathscr{M}$ compactness and nano $\mathscr{M}$ connectedness and also dealt with it.

Keywords
$\mathfrak{N}\mathscr{M}T_0$, $\mathfrak{N}\mathscr{M}T_1$, $\mathfrak{N}\mathscr{M}T_2$, $\mathfrak{N}\mathscr{M}reg$, $\mathfrak{N}\mathscr{M}nor$ spaces, nano $\mathscr{M}$ compactness and nano $\mathscr{M}$ connectedness.

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Contents

1. Introduction and Preliminaries

Lellis Thivagar [4] introduced the notion of Nano topology (briefly, $\mathfrak{N}\mathfrak{T}$) by using theory approximations and boundary region of a subset of an universe in terms of an equivalence relation on it and also defined Nano closed (briefly, $\mathfrak{N}c$) sets , Nano-interior (briefly, $\mathfrak{N}int$) and Nano-closure (briefly, $\mathfrak{N}cl$) in a nano topological spaces (briefly, $\mathfrak{N}ts$). Carmel [10] discussed some weak forms of $\mathfrak{N}o$ sets and $\mathfrak{N}\theta$ open (briefly, $\mathfrak{N}\theta o$) sets. The notion of M-open sets in topological spaces were introduced by El-Maghrabi and Al-Juhani [1] in 2011 and studied some of their properties. The class of sets namely M-open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. Khalaf and Ahmed Elmoasry [2] discussed slightly nano separation axioms in $\mathfrak{N}ts$'s. By these motivations, we present the concept of nano M-open sets [5] and study their properties and applications in nano topological space. The purpose of this paper is to discuss the separation axioms via $\mathfrak{N}\mathscr{M}o$ sets such as $\mathfrak{N}\mathscr{M}T_0$, $\mathfrak{N}\mathscr{M}T_1$, $\mathfrak{N}\mathscr{M}T_2$, $\mathfrak{N}\mathscr{M}reg$ and $\mathfrak{N}\mathscr{M}nor$ spaces are introduced and some of the properties are discussed. Further, the concept of nano $\mathscr{M}$ compactness and nano $\mathscr{M}$ connectedness by using $\mathfrak{N}\mathscr{M}o$ sets in $\mathfrak{N}ts$'s are introduced and investigated some of their properties.

Definition 1.1. [10] Let $(U,\tau_R(P))$ be a $\mathfrak{N}ts$ and let $A\subseteq U$ then the nano θ-interior (resp. nano θ-closure) of A is defined and denoted by $\mathfrak{N}int_\theta(A)=\bigcup\{B : B$ is a $\mathfrak{N}o$ set and $\mathfrak{N}cl(B)\subseteq A\}$ (resp. $\mathfrak{N}cl_\theta(A)=\bigcup\{l\in U : \mathfrak{N}cl(B)\cap A\neq\phi,\ B$ is a $\mathfrak{N}o$ set and $l\in B\}$.

Definition 1.2. [10] A subset A of P is said to be nano θ-open (resp. nano θ-closed) (briefly, $\mathfrak{N}\theta o$ (resp. $\mathfrak{N}\theta c$)) set if $A=\mathfrak{N}int_\theta(A)$ (resp. A^c is a nano θ-open set).

Definition 1.3. [4, 9] Let $(U,\tau_R(P))$ be a $\mathfrak{N}ts$ and $A\subseteq U$. Then A is said to be nano regular open (briefly, $\mathfrak{N}ro$) if $A=\mathfrak{N}int(\mathfrak{N}cl(A))$.

Definition 1.4. [7] Let $(U,\tau_R(P))$ be a $\mathfrak{N}ts$ and let $A\subseteq U$ then the nano δ-interior (resp. nano δ-closure) of A is defined and denoted by $\mathfrak{N}int_\delta(A)=\bigcup\{B : B$ is a $\mathfrak{N}ro$ set and $B\subseteq A\}$ (resp. $\mathfrak{N}cl_\delta(A)=\bigcup\{l\in U : \mathfrak{N}int(\mathfrak{N}cl(B))\cap A\neq\phi,\ B$ is a $\mathfrak{N}o$ set and $l\in B\}$.

Definition 1.5. [7] A subset A of P is said to be nano δ-open (resp. nano δ-closed) (briefly, $\mathfrak{N}\delta o$ (resp. $\mathfrak{N}\delta c$)) set if $A=\mathfrak{N}int_\delta(A)$ (resp. A^c is a nano δ-open set).

Definition 1.6. [5] Let $(U, \tau_R(P))$ be a $\mathfrak{N}ts$ and $A \subseteq U$. Then A is said to be a nano M-open (resp. nano M-closed) set (briefly $\mathfrak{N}\mathscr{M}o$ (resp. $\mathfrak{N}\mathscr{M}c$)) if $A \subseteq \mathfrak{N}cl(\mathfrak{N}int_\theta\ (A) \cup \mathfrak{N}int(\mathfrak{N}cl_\delta(A))$ (resp. $A \supseteq \mathfrak{N}int(\mathfrak{N}cl_\theta(A)) \cap \mathfrak{N}cl(\mathfrak{N}\ int_\delta(A)))$.

The family of all $\mathfrak{N}\mathscr{M}o$ (resp. $\mathfrak{N}\mathscr{M}c$) sets are denoted by $\mathfrak{N}\mathscr{M}O(U, \tau_R(P))$, (resp. $\mathfrak{N}\mathscr{M}C(U, \tau_R(P))$).

Definition 1.7. [5] Let $(U, \tau_R(P))$ be a $\mathfrak{N}ts$ and let $A \subseteq U$ then the nano M-interior of A is the union of all $\mathfrak{N}\mathscr{M}o$ sets contained in A and denoted by $\mathfrak{N}\mathscr{M}int(A)$.

Definition 1.8. [5] Let $(U, \tau_R(P))$ be a $\mathfrak{N}ts$ and let $A \subseteq U$ then the nano M-closure of A is the intersection of all $\mathfrak{N}\mathscr{M}c$ sets containing A and denoted by $\mathfrak{N}\mathscr{M}cl(A)$.

Definition 1.9. [6] A function $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q\ ))$ is said to be Nano $\mathscr{M}$ continuous (briefly, $\mathfrak{N}\mathscr{M}$-Cts), if for each $\mathfrak{N}c$ set K of V, the set $f^{-1}(K)$ is $\mathfrak{N}\mathscr{M}c$ set of U.

Definition 1.10. [6] A function $f : (U, \tau_R(P)) \to (V, \sigma_{R'}\ (Q))$ is called Nano $\mathscr{M}$ irresolute (briefly, $\mathfrak{N}\mathscr{M}Irr$) function, if for each $\mathfrak{N}\mathscr{M}c$ subset K of V, the set $f^{-1}(K)$ is $\mathfrak{N}\mathscr{M}c$ subset of U.

Definition 1.11. [6] A function $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}\ (Q))$ is said to be Nano $\mathscr{M}$ open function (briefly, $\mathfrak{N}\mathscr{M}of$) if the direct image $f(K)$ is $\mathfrak{N}\mathscr{M}o$ set in V whenever K is $\mathfrak{N}o$ in U.

Definition 1.12. [11] A collection $\{K_i : i \in J\}$ of $\mathfrak{N}o$ sets in $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano open cover (briefly, $\mathfrak{N}ocov$) of a subset B of $(U, \tau_R(P))$, if $B \subset \bigcup\{K_i : i \in J\}$.

Definition 1.13. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano compact (briefly, $\mathfrak{N}comp$) if every $\mathfrak{N}ocov$ of U has a finite subcover.

Definition 1.14. [11] A subset B of $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called $\mathfrak{N}comp$ relative to U if for every collection $\{K_i : i \in J\}$ of $\mathfrak{N}o$ subsets of U such that $B \subset \bigcup\{K_i : i \in J\}$, there exists a finite subset J_0 of J such that $B \subset \bigcup\{K_i : i \in J_0\}$.

Definition 1.15. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano connected (briefly, $\mathfrak{N}con$) if U is not the disjoint union of two nonempty $\mathfrak{N}o$ subsets.

Definition 1.16. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano -T_0 (briefly, $\mathfrak{N}T_0$) space, if for any two points $l \neq m$, there exists a $\mathfrak{N}o$ set containing one of them but not the other.

Definition 1.17. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano -T_1 (briefly, $\mathfrak{N}T_1$) space, if for any two points $l \neq m$, there exists a $\mathfrak{N}\mathscr{M}o$ sets G and H with $l \in G$, $m \notin G$ and $l \notin H$, $m \in H$.

Definition 1.18. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano -T_2 (briefly, $\mathfrak{N}T_2$) space, if for any two points $l \neq m$, there exists disjoint $\mathfrak{N}o$ sets G and H with $l \in G$ and $m \in H$.

Definition 1.19. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano regular (briefly, $\mathfrak{N}reg$) space, if for any point l and $\mathfrak{N}c$ set F not containing l, there exists disjoint $\mathfrak{N}o$ sets G and H with $l \in G$ and $F \subset H$.

Definition 1.20. [11] A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano normal (briefly, $\mathfrak{N}nor$) space, if for any two disjoint $\mathfrak{N}c$ sets E and F, there exists disjoint $\mathfrak{N}o$ sets G and H with $E \subset G$ and $F \subset H$.

Throughout this paper, $(U, \tau_R(P))$ is a $\mathfrak{N}ts$ with respect to P where $P \subseteq U$, R is an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R. All other undefined notions from [3, 4, 8].

2. Nano $\mathscr{M}$ Separation Axioms

Definition 2.1. A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano $\mathscr{M}$-T_0 (briefly, $\mathfrak{N}\mathscr{M}T_0$) space, if for any two points $l \neq m$, there exists a $\mathfrak{N}\mathscr{M}o$ set containing one of them but not the other.

Clearly, every $\mathfrak{N}T_0$ space is $\mathfrak{N}\mathscr{M}T_0$ space.

Theorem 2.2. Let $\mathfrak{N}\mathscr{M}C(U,P)$ is closed under arbitrary intersection and a $\mathfrak{N}ts$ $(U, \tau_R(P))$ is a $\mathfrak{N}\mathscr{M}T_0$ space iff $\mathfrak{N}\mathscr{M}$ closures of distinct points are distinct.

Proof. Let l, m be distinct points of U. Since U is a $\mathfrak{N}\mathscr{M}T_0$ space, there exists a $\mathfrak{N}\mathscr{M}o$ set G such that $l \in G$ and $m \notin G$. Consequently, $U - G$ is a $\mathfrak{N}\mathscr{M}c$ set containing m but not l. But $\mathfrak{N}\mathscr{M}cl(\{m\})$ is the intersection of all $\mathfrak{N}\mathscr{M}c$ sets containing m. Hence $m \in \mathfrak{N}\mathscr{M}cl(\{m\})$ but $l \notin \mathfrak{N}\mathscr{M}cl(\{m\})$ as $l \notin U - G$. Therefore $\mathfrak{N}\mathscr{M}cl(\{l\}) \neq \mathfrak{N}\mathscr{M}cl(\{m\})$.

Conversely, let $\mathfrak{N}\mathscr{M}cl(\{l\}) \neq \mathfrak{N}\mathscr{M}cl(\{m\})$ for $l \neq m$. Then there exist atleast one point $n \in U$ such that $n \in \mathfrak{N}\mathscr{M}cl(\{l\})$ but $n \notin \mathfrak{N}\mathscr{M}cl(\{m\})$. Suppose $l \notin \mathfrak{N}\mathscr{M}cl(\ \{m\})$ because if $l \in \mathfrak{N}\mathscr{M}cl(\{m\})$ then $\{l\} \subset \mathfrak{N}\mathscr{M}cl(\{m\})$ implies $\mathfrak{N}\mathscr{M}cl(\{l\}) \subset \mathfrak{N}\mathscr{M}cl(\{m\})$. So $n \in \mathfrak{N}\mathscr{M}cl(\{m\})$ which is a contradiction. Hence $l \notin \mathfrak{N}\mathscr{M}cl(\{m\})$ which implies $l \in U - \mathfrak{N}\mathscr{M}cl(\{m\})$ which is a $\mathfrak{N}\mathscr{M}o$ set containing l but not m. Hence U is a $\mathfrak{N}\mathscr{M}T_0$ space.

Theorem 2.3. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is a injective, $\mathfrak{N}\mathscr{M}irr$ function and V is a $\mathfrak{N}\mathscr{M}T_0$ then U is a $\mathfrak{N}\mathscr{M}T_0$.

Proof. Suppose V is a $\mathfrak{N}\mathscr{M}T_0$ space. Let l and m be any two distinct points in U. Since f is injective $f(l)$ and $f(m)$ are distinct points in V. Since V is a $\mathfrak{N}\mathscr{M}T_0$ space, there exists a $\mathfrak{N}\mathscr{M}o$ set G in V containing $f(l)$ but not $f(m)$. Again since f is $\mathfrak{N}\mathscr{M}irr$, $f^{-1}(G)$ is a $\mathfrak{N}\mathscr{M}o$ set in U containing l but not m. Therefore U is a $\mathfrak{N}\mathscr{M}T_0$ space.

Theorem 2.4. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is a injective, $\mathfrak{N}\mathscr{M}Cts$ function and V is a $\mathfrak{N}T_0$ space then U is a $\mathfrak{N}\mathscr{M}T_0$ space.

Proof. Let l and m be any two distinct points in U. Since f is injective $f(l)$ and $f(m)$ are distinct points in V. Since V is a $\mathfrak{N}T_0$ space, there exists a $\mathfrak{N}o$ set G in V containing $f(l)$ but not $f(m)$. Again since f is $\mathfrak{N}\mathscr{M}Cts$, $f^{-1}(G)$ is a $\mathfrak{N}\mathscr{M}o$ set in U containing l but not m. Therefore U is a $\mathfrak{N}\mathscr{M}T_0$ space.

Definition 2.5. A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano $\mathscr{M}$-T_1 (briefly, $\mathfrak{N}\mathscr{M}T_1$) space, if for any two points $l \neq m$, there exists a $\mathfrak{N}\mathscr{M}o$ sets G and H with $l \in G$, $m \notin G$ and $l \notin H$, $m \in H$.

Clearly, every $\mathfrak{N}T_1$ space is $\mathfrak{N}\mathscr{M}T_1$ space.

Theorem 2.6. Let $\mathfrak{N}\mathscr{M}O(U,P)$ is closed under arbitrary union then a $\mathfrak{N}ts$ $(U,\tau_R(P))$ is a $\mathfrak{N}\mathscr{M}T_1$ space iff every singleton subset $\{l\}$ of U is a $\mathfrak{N}\mathscr{M}c$ set.

Proof. Let U be a $\mathfrak{N}\mathscr{M}T_1$ space and $l \in U$. Let $m \in U-\{l\}$. Then for $l \neq m$ there exist a $\mathfrak{N}\mathscr{M}o$ set G_m such that $m \in G_m$ and $l \notin G_m$. Consequently, $m \in G_m \subset U-\{l\}$. That is $U-\{l\} = \bigcup\{G_m : m \in U-\{l\}\}$, which is the union of $\mathfrak{N}\mathscr{M}o$ sets and hence $\mathfrak{N}\mathscr{M}o$. Therefore $\{l\}$ is a $\mathfrak{N}\mathscr{M}c$ set.

Conversely, suppose $\{l\}$ is a $\mathfrak{N}\mathscr{M}c$ set for every $l \in U$. Let l and $m \in U$ with $l \neq m$. Now $l \neq m$ implies $m \in U-\{l\}$. Hence $U-\{l\}$ is a $\mathfrak{N}\mathscr{M}o$ set containing m but not l. Similarly $U-\{m\}$ is a $\mathfrak{N}\mathscr{M}o$ set containing l but not m. Hence U is a $\mathfrak{N}\mathscr{M}T_1$ space.

Theorem 2.7. If $f\colon (U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is a injective, $\mathfrak{N}\mathscr{M}irr$ function and V is a $\mathfrak{N}\mathscr{M}T_1$ space then U is also a $\mathfrak{N}\mathscr{M}T_1$ space.

Proof. Let l_1 and l_2 be pair of distinct points in U. Since f is injective, there exist two distinct points m_1 and m_2 of V such that $f(l_1) = m_1$ and $f(l_2) = m_2$. Since V is a $\mathfrak{N}\mathscr{M}T_1$ space, there exist a $\mathfrak{N}\mathscr{M}o$ sets G and H in V such that $m_1 \in G$, $m_2 \notin G$ and $m_1 \notin H$, $m_2 \in H$. That is $l_1 \in f^{-1}(G)$, $l_1 \notin f^{-1}(H)$ and $l_2 \in f^{-1}(H)$, $l_2 \notin f^{-1}(G)$. Since f is $\mathfrak{N}\mathscr{M}irr$ function, $f^{-1}(G)\&\ f^{-1}(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Thus for two distinct points l_1 and l_2 of U there exists $\mathfrak{N}\mathscr{M}o$ sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$, $l_1 \notin f^{-1}(H)$ and $l_2 \in f^{-1}(H)$, $l_2 \notin f^{-1}(G)$. Therefore U is $\mathfrak{N}\mathscr{M}T_1$ space.

Theorem 2.8. If $f\colon (U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is a $\mathfrak{N}\mathscr{M}$ Cts injection and V is a $\mathfrak{N}T_1$ space then U is a $\mathfrak{N}\mathscr{M}T_1$ space.

Proof. For any two distinct points l_1 and l_2 of U, there exist two distinct points m_1 and m_2 of V such that $f(l_1) = m_1$ and $f(l_2) = m_2$. Since V is a $\mathfrak{N}T_1$ space, there exist a $\mathfrak{N}o$ sets G and H in V such that $m_1 \in G$, $m_2 \notin G$ and $m_1 \notin H$, $m_2 \in H$. That is $l_1 \in f^{-1}(G)$, $l_1 \notin f^{-1}(H)$ and $l_2 \in f^{-1}(H)$, $l_2 \notin f^{-1}(G)$. Since f is a $\mathfrak{N}\mathscr{M}Cts$ function, $f^{-1}(G)\&\ f^{-1}(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Thus for two distinct points l_1 and l_2 of U there exists $\mathfrak{N}\mathscr{M}o$ sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$, $l_1 \notin f^{-1}(H)$ and $l_2 \in f^{-1}(H)$, $l_2 \notin f^{-1}(G)$. Therefore U is $\mathfrak{N}\mathscr{M}T_1$ space.

Definition 2.9. A $\mathfrak{N}ts$ $(U,\tau_R(P))$ is called Nano $\mathscr{M}$-T_2 (briefly, $\mathfrak{N}\mathscr{M}T_2$) space, if for any two points $l \neq m$, there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H with $l \in G$ and $m \in H$.

Clearly, every $\mathfrak{N}T_2$ space is $\mathfrak{N}\mathscr{M}T_2$.

Theorem 2.10. If $f\colon (U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is a $\mathfrak{N}\mathscr{M}$ Cts injection and V is a $\mathfrak{N}T_2$ space then U is a $\mathfrak{N}\mathscr{M}T_2$ space.

Proof. For any two distinct points l_1 and l_2 of U, there exist two distinct points m_1 and m_2 of V such that $f(l_1) = m_1$ and $f(l_2) = m_2$. Since V is a $\mathfrak{N}T_2$ space, there exist a disjoint $\mathfrak{N}o$ sets G and H in V such that $m_1 \in G$ and $m_2 \in H$. That is $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$. Since f is a $\mathfrak{N}\mathscr{M}Cts$ function, $f^{-1}(G)\&\ f^{-1}(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Further as f is injective $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$. Thus for two distinct points l_1 and l_2 of U there exists $\mathfrak{N}\mathscr{M}o$ sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$. Therefore U is $\mathfrak{N}\mathscr{M}T_2$ space.

Theorem 2.11. If $f\colon (U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is a injective, $\mathfrak{N}\mathscr{M}irr$ function and V is a $\mathfrak{N}\mathscr{M}T_2$ space then U is also a $\mathfrak{N}\mathscr{M}T_2$ space.

Proof. Let l_1 and l_2 be a pair of distinct points in U. Since f is injective, there exist two distinct points m_1 and m_2 of V such that $f(l_1) = m_1$ and $f(l_2) = m_2$. Since V is a $\mathfrak{N}\mathscr{M}T_2$ space, there exist a disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H in V such that $m_1 \in G$ and $m_2 \in H$. That is $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$. Since f is a $\mathfrak{N}\mathscr{M}irr$ function, $f^{-1}(G)\&\ f^{-1}(H)$ are disjoint $\mathfrak{N}\mathscr{M}o$ sets in U. Thus for two distinct points l_1 and l_2 of U there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$. Therefore U is $\mathfrak{N}\mathscr{M}T_2$ space.

Theorem 2.12. A $\mathfrak{N}ts$ $(U,\tau_R(P))$ is a $\mathfrak{N}\mathscr{M}T_2$ space iff for each $l \neq m$, then there exists a $\mathfrak{N}\mathscr{M}o$ set G such that $l \in G$ and $m \notin \mathfrak{N}\mathscr{M}cl(G)$.

Proof. Assume that U is a $\mathfrak{N}\mathscr{M}T_2$ space. Let $l,\ m \in U$ and $l \neq m$, then there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H such that $l \in G$ and $m \in H$. Clearly, $U-H$ is $\mathfrak{N}\mathscr{M}c$ set. Since $G \cap H = \emptyset$, $G \subset U-H$. Therefore $\mathfrak{N}\mathscr{M}cl(G) \subset \mathfrak{N}\mathscr{M}cl(U-H) = U-H$. Now $m \notin U-H$ implies $m \notin \mathfrak{N}\mathscr{M}cl(G)$.

Conversely, let $l,\ m \in U$ and $l \neq m$. By hypothesis there exists a $\mathfrak{N}\mathscr{M}o$ set G such that $l \in G$ and $m \notin \mathfrak{N}\mathscr{M}cl(G)$. This implies there exists a $\mathfrak{N}\mathscr{M}c$ set H such that $m \notin H$. Therefore $m \in U-H$ is a $\mathfrak{N}\mathscr{M}o$ set. Thus there exists two disjoint $\mathfrak{N}\mathscr{M}o$ sets G and $U-H$ such that $l \in G$ and $m \in U-H$. Therefore U is a $\mathfrak{N}\mathscr{M}T_2$ space.

3. Nano $\mathscr{M}$ Regular Spaces

Definition 3.1. A $\mathfrak{N}ts$ $(U,\tau_R(P))$ is called Nano $\mathscr{M}$ regular (briefly, $\mathfrak{N}\mathscr{M}reg$) space, if for any point l and $\mathfrak{N}c$ set F not containing l, there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H with $l \in G$ and $F \subset H$.

Clearly, every $\mathfrak{N}reg$ space is $\mathfrak{N}\mathscr{M}reg$ space.

Theorem 3.2. If $f\colon (U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is $\mathfrak{N}Cts$ bijective, $\mathfrak{N}\mathscr{M}o$ function and U is a $\mathfrak{N}reg$ space then V is $\mathfrak{N}\mathscr{M}reg$.

Proof. Let F be a $\mathfrak{N}c$ set in V and $m \notin F$. Take $m = f(l)$ for some $l \in U$. Since f is $\mathfrak{N}Cts$, $f^{-1}(F)$ is $\mathfrak{N}c$ in U such that $l \notin f^{-1}(F)$. Now U is a $\mathfrak{N}reg$ space, there exist disjoint $\mathfrak{N}o$ sets G and H such that $l \in G$ and $f^{-1}(F) \subset H$. That is $m = f(l) \in f(G)$ and $F \subset f(H)$. Since f is $\mathfrak{N}\mathscr{M}of$, $f(G)\&\ f(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in V and f is bijective, $f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$. Therefore V is a $\mathfrak{N}\mathscr{M}reg$ space.

Theorem 3.3. If $f\colon (U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}Cts$, $\mathfrak{N}c$ injection and V is a $\mathfrak{N}reg$ space then U is $\mathfrak{N}\mathscr{M}reg$.

Proof. Let F be a $\mathfrak{N}c$ set in U and $l \notin F$. Since f is $\mathfrak{N}c$ injection, $f(F)$ is $\mathfrak{N}c$ in V such that $f(l) \notin f(F)$. Now V is a $\mathfrak{N}reg$ space, there exist disjoint $\mathfrak{N}o$ sets G and H such that $f(l) \in G$ and $f(F) \subset H$. This implies $l \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $\mathfrak{N}\mathscr{M}Cts$ function, $f^{-1}(G)\& f^{-1}(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Further, $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence U is a $\mathfrak{N}\mathscr{M}reg$ space.

Theorem 3.4. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}irr$, $\mathfrak{N}c$ injection and V is a $\mathfrak{N}\mathscr{M}reg$ space then U is also a $\mathfrak{N}\mathscr{M}reg$ space.

Proof. Let F be a $\mathfrak{N}c$ set in U and $l \notin F$. Since f is $\mathfrak{N}c$ injection, $f(F)$ is $\mathfrak{N}c$ in V such that $f(l) \notin f(F)$. Now V is a $\mathfrak{N}\mathscr{M}reg$ space, there exist disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H such that $f(l) \in G$ and $f(F) \subset H$. This implies $l \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $\mathfrak{N}\mathscr{M}irr$, $f^{-1}(G)\& f^{-1}(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Further, $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence U is a $\mathfrak{N}\mathscr{M}reg$ space.

Theorem 3.5. Let $\mathfrak{N}\mathscr{M}C(U,P)$ is closed under arbitrary intersection and in any $\mathfrak{N}ts$ $(U, \tau_R(P))$ the following are equivalent.

(i) U is $\mathfrak{N}\mathscr{M}reg$.

(ii) For every point $l \in U$ and $\mathfrak{N}o$ set G containing l there exists $\mathfrak{N}\mathscr{M}o$ set H such that $l \in H \subset \mathfrak{N}\mathscr{M}cl\ (H) \subset G$.

(iii) For every $\mathfrak{N}c$ set F, $F = \cap\{\mathfrak{N}\mathscr{M}cl(G) : F \subset G$ and $G \in \mathfrak{N}\mathscr{M}O(U,P)\}$.

(iv) For every set A and a $\mathfrak{N}o$ set B such that $A \cap B \neq \emptyset$, there exists $\mathfrak{N}\mathscr{M}o$ set O such that $A \cap O \neq \emptyset$ and $\mathfrak{N}\mathscr{M}cl(O) \subset B$.

(v) For every non empty set A and $\mathfrak{N}c$ set B such that $A \cap B \neq \emptyset$, there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets L and M such that $A \cap L \neq \emptyset$ and $B \subset M$.

Proof. (i) $\Rightarrow$ (ii): Let G be a $\mathfrak{N}o$ set containing l. Then $U - G$ is $\mathfrak{N}c$ set not containing l. Since U is $\mathfrak{N}\mathscr{M}reg$, there exists $\mathfrak{N}\mathscr{M}o$ sets S and H such that $l \in H$, $U - G \subset S$ and $H \cap S = \emptyset$. This implies $H \subset U - S$. Therefore $\mathfrak{N}\mathscr{M}cl(H) \subset \mathfrak{N}\mathscr{M}cl(U - S) = U - S$, because $U - S$ is $\mathfrak{N}\mathscr{M}c$. Hence $l \in H \subset \mathfrak{N}\mathscr{M}cl(H) \subset U - S \subset G$. That is $l \in \mathfrak{N}\mathscr{M}cl(H) \subset G$.

(ii) $\Rightarrow$ (iii): Let F be a $\mathfrak{N}c$ set and $l \notin F$. Then $U - F$ is a $\mathfrak{N}o$ set containing l. By (ii) there is a $\mathfrak{N}\mathscr{M}o$ set H such that $l \in H \subset \mathfrak{N}\mathscr{M}cl(H) \subset U - F$. And so, $F \subset U - \mathfrak{N}\mathscr{M}cl(H) \subset U - H$. Consequently, $U - H$ is $\mathfrak{N}\mathscr{M}c$ set not containing l. Put $G = U - \mathfrak{N}\mathscr{M}cl(H)$. This implies $F \subset G$ and G is $\mathfrak{N}\mathscr{M}o$ set of U and $l \notin \mathfrak{N}\mathscr{M}cl(G)$, implies $\cap\{\mathfrak{N}\mathscr{M}cl(G) : F \subset G$ and $G \in \mathfrak{N}\mathscr{M}O(U,P)\} \subset F$. But F is $\mathfrak{N}c$ and every $\mathfrak{N}c$ set is $\mathfrak{N}\mathscr{M}c$. Therefore $F \subset \cap\{\mathfrak{N}\mathscr{M}cl(G) : F \subset G$ and $G \in \mathfrak{N}\mathscr{M}O(U,P)\}$ is always true. Thus $F = \cap\{\mathfrak{N}\mathscr{M}cl(G) : F \subset G \& G \in \mathfrak{N}\mathscr{M}O(U,P)\}$.

(iii) $\Rightarrow$ (iv): Let $A \cap B \neq \emptyset$ and B is $\mathfrak{N}o$. Let $l \in A \cap B$. Then $U - B$ is a $\mathfrak{N}c$ set not containing l. By (iii) there exists a $\mathfrak{N}\mathscr{M}o$ set G of U such that $U - B \subset G$ and $l \notin \mathfrak{N}\mathscr{M}cl(G)$. Put $O = U - \mathfrak{N}\mathscr{M}cl(G)$, then O is $\mathfrak{N}\mathscr{M}o$ set of U, $l \in A \cap O$ and $\mathfrak{N}\mathscr{M}cl(O) \subset \mathfrak{N}\mathscr{M}cl(U - G) = U - G \subset B$. Hence $\mathfrak{N}\mathscr{M}cl(O) \subset B$.

(iv) $\Rightarrow$ (v): If $A \cap B \neq \emptyset$, where A is non empty and B is $\mathfrak{N}c$, then $A \cap (U - B) \neq \emptyset$ and $U - B$ is $\mathfrak{N}o$. Therefore by (iv) there exists a $\mathfrak{N}\mathscr{M}o$ set L such that $A \cap L \neq \emptyset$ and $L \subset \mathfrak{N}\mathscr{M}cl(L) \subset U - B$. Put $U - \mathfrak{N}\mathscr{M}cl(L) = M$, then M is $\mathfrak{N}\mathscr{M}o$ set of U, $U - B \subset L$ and $\mathfrak{N}\mathscr{M}cl(U - L) \subset M, U - L \subset M$. Hence $B \subset M$.

(v) $\Rightarrow$ (i): Let F be a $\mathfrak{N}c$ set such that $l \notin F$, then $\{l\} \cap F = \emptyset$. By (v), there exist disjoint $\mathfrak{N}o$ sets L and M such that $\{l\} \cap L = \emptyset$ and $F \subset M$, and hence U is $\mathfrak{N}\mathscr{M}reg$.

4. Nano $\mathscr{M}$ Normal Spaces

Definition 4.1. A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano $\mathscr{M}$ normal (briefly, $\mathfrak{N}\mathscr{M}nor$) space, if for any two disjoint $\mathfrak{N}c$ sets E and F, there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H with $E \subset G$ and $F \subset H$.

Clearly, every $\mathfrak{N}nor$ space is $\mathfrak{N}\mathscr{M}nor$ space.

Theorem 4.2. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}Cts$ bijective, $\mathfrak{N}\mathscr{M}o$ function from a $\mathfrak{N}nor$ space U on to a $\mathfrak{N}ts$ V then V is $\mathfrak{N}\mathscr{M}nor$.

Proof. Let E and F be disjoint $\mathfrak{N}c$ sets in V. Since f is $\mathfrak{N}Cts$ bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint $\mathfrak{N}c$ sets in U. Now U is a $\mathfrak{N}nor$ space, there exist disjoint $\mathfrak{N}o$ sets G and H such that $f^{-1}(E) \subset G$ and $f^{-1}(F) \subset H$. That is $E \subset f(G)$ and $F \subset f(H)$. Since f is $\mathfrak{N}\mathscr{M}of$, $f(G)\& f(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in V and f is injective, $f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$. Therefore V is a $\mathfrak{N}\mathscr{M}nor$ space.

Theorem 4.3. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}Cts$, $\mathfrak{N}c$ injection and V is a $\mathfrak{N}nor$ space then U is $\mathfrak{N}\mathscr{M}nor$.

Proof. Let E and F be disjoint $\mathfrak{N}c$ sets in V. Since f is $\mathfrak{N}c$ injection, $f(E)$ and $f(F)$ are disjoint $\mathfrak{N}c$ sets in V. Now V is a $\mathfrak{N}nor$ space, there exist disjoint $\mathfrak{N}o$ sets G and H such that $f(E) \subset G$ and $f(F) \subset H$. That is $E \subset f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $\mathfrak{N}\mathscr{M}Cts$ function, $f^{-1}(G)\& f^{-1}(H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Therefore U is a $\mathfrak{N}\mathscr{M}nor$ space.

Theorem 4.4. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}irr$, $\mathfrak{N}c$ injection and V is a $\mathfrak{N}\mathscr{M}nor$ then U is $\mathfrak{N}\mathscr{M}nor$.

Proof. Let E and F be disjoint $\mathfrak{N}c$ sets in V. Since f is $\mathfrak{N}c$ injection, $f(E)$ and $f(F)$ are disjoint $\mathfrak{N}c$ sets in V. Now V is a $\mathfrak{N}nor$ space, there exist disjoint $\mathfrak{N}\mathscr{M}o$ sets G and H such that $f(E) \subset G$ and $f(F) \subset H$. That is $E \subset f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $\mathfrak{N}\mathscr{M}irr$, $f^{-1}(G)\& f^{-1}\ (H)$ are $\mathfrak{N}\mathscr{M}o$ sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Therefore U is a $\mathfrak{N}\mathscr{M}nor$ space.

Theorem 4.5. The following statements are equivalent for a $\mathfrak{N}ts$ $(U, \tau_R(P))$.

(i) U is $\mathfrak{N}\mathscr{M}nor$.

(ii) For each $\mathfrak{N}c$ set A and for each $\mathfrak{N}o$ set G containing A, there exists a $\mathfrak{N}\mathscr{M}o$ set S containing A such that $\mathfrak{N}\mathscr{M}cl(S) \subset G$

(iii) For each pair of disjoint $\mathfrak{N}c$ sets A and B there exists a $\mathfrak{N}\mathscr{M}o$ set G containing A such that $\mathfrak{N}\mathscr{M}cl(G) \cap B = \emptyset$.

Proof. (i) $\Rightarrow$ (ii): Let A be a $\mathfrak{N}c$ set and G be a $\mathfrak{N}o$ set containing A. Then $A \cap (U-G) = \emptyset$. Therefore they are disjoint $\mathfrak{N}c$ sets in U. Since U is $\mathfrak{N}\mathscr{M}nor$, there exists disjoint $\mathfrak{N}\mathscr{M}o$ sets S and H such that $A \subset S$, $U-G \subset H$ that is $U-H \subset G$. Now $S \cap H = \emptyset$, implies $S \subset U-H$. Therefore $\mathfrak{N}\mathscr{M}cl(S) \subset \mathfrak{N}\mathscr{M}cl(U-H) = U-H$, because $U-H$ is $\mathfrak{N}\mathscr{M}c$ set. Thus, $A \subset S \subset \mathfrak{N}\mathscr{M}cl(S) = U-H \subset G$, That is $\mathfrak{N}\mathscr{M}cl(S) \subset G$.

(ii) $\Rightarrow$ (iii): Let A and B be disjoint $\mathfrak{N}c$ sets in U, then $A \subset U-B$ and $U-B$ is $\mathfrak{N}o$ set containing A. By (ii) there exists $\mathfrak{N}\mathscr{M}o$ set G such that $A \subset G$ and $\mathfrak{N}\mathscr{M}cl(G) \subset U-B$, which implies $\mathfrak{N}\mathscr{M}cl(G) \cap B = \emptyset$.

(iii) $\Rightarrow$ (i): Let A and B be disjoint $\mathfrak{N}c$ sets in U. By (iii) there exists $\mathfrak{N}\mathscr{M}o$ set G such that $A \subset G$ and $\mathfrak{N}\mathscr{M}cl(G) \cap B = \emptyset$ or $B \subset U - \mathfrak{N}\mathscr{M}cl(G)$. Now G and $U - \mathfrak{N}\mathscr{M}cl(G)$ are disjoint $\mathfrak{N}\mathscr{M}o$ sets of U such that $A \subset G$ and $B \subset U - \mathfrak{N}\mathscr{M}cl(G)$. Hence U is $\mathfrak{N}\mathscr{M}nor$.

5. Nano $\mathscr{M}$ Compactness

Definition 5.1. A collection $\{K_i : i \in J\}$ of $\mathfrak{N}\mathscr{M}o$ sets in $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano $\mathscr{M}$ open cover (briefly, $\mathfrak{N}\mathscr{M}ocov$) of a subset L of $(U, \tau_R(P))$, if $L \subset \bigcup\{K_i : i \in J\}$.

Definition 5.2. A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano $\mathscr{M}$ compact (briefly, $\mathfrak{N}\mathscr{M}comp$) if every $\mathfrak{N}\mathscr{M}ocov$ of U has a finite subcover.

Definition 5.3. A subset L of $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called $\mathfrak{N}\mathscr{M}comp$ relative to U if for every collection $\{K_i : i \in J\}$ of $\mathfrak{N}\mathscr{M}o$ subsets of U such that $L \subset \bigcup\{K_i : i \in J\}$, there exists a finite subset J_0 of J such that $L \subset \bigcup\{K_i : i \in J_0\}$

Definition 5.4. A subset L of $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called $\mathfrak{N}\mathscr{M}comp$ if L is $\mathfrak{N}\mathscr{M}comp$ as a subspace of U.

Theorem 5.5. Every $\mathfrak{N}\mathscr{M}c$ subset of $\mathfrak{N}\mathscr{M}comp$ space is $\mathfrak{N}\mathscr{M}c$ relative to U, if $\mathfrak{N}\mathscr{M}O(U,P)$ is closed under arbitrary union.

Proof. Let U be $\mathfrak{N}\mathscr{M}comp$ space and K is $\mathfrak{N}\mathscr{M}c$ subset of U. Then $U-K$ is $\mathfrak{N}\mathscr{M}o$ in U. Let $S = \{K_i : i \in J\}$ be $\mathfrak{N}\mathscr{M}ocov$ of K by $\mathfrak{N}\mathscr{M}o$ subsets in U. Then $S^* = S \cup (U-K)$ is a $\mathfrak{N}\mathscr{M}ocov$ of U. Since U is $\mathfrak{N}\mathscr{M}comp$, S^* is reducible to a finite subcover of U say $U = K_{i_1} \cup K_{i_2} \cup \ldots \cup K_{i_n}, K_{i_k} \in S$. But K and $U-K$ are disjoint. Hence $K \subset K_{i_1} \cup K_{i_2} \cup \ldots \cup K_{i_n} \in S$. This implies that any $\mathfrak{N}\mathscr{M}ocov$ S of K contains a finite subcover. Therefore K is $\mathfrak{N}\mathscr{M}comp$ relative to U.

Theorem 5.6. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is surjective, $\mathfrak{N}\mathscr{M}Cts$ function and U is $\mathfrak{N}\mathscr{M}comp$ then V is $\mathfrak{N}comp$.

Proof. Let $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is surjective, $\mathfrak{N}\mathscr{M}Cts$ from $\mathfrak{N}\mathscr{M}comp$ space $(U, \tau_R(P))$ to $\mathfrak{N}ts$ $(V, \sigma_{R'}(Q))$. Let $\{K_i : i \in J\}$ be $\mathfrak{N}ocov$ of V. Since f is $\mathfrak{N}\mathscr{M}Cts$, $\{f^{-1}(K_i) : i \in J\}$ is $\mathfrak{N}\mathscr{M}ocov$ of U. Since U is $\mathfrak{N}\mathscr{M}comp$ implies $\mathfrak{N}\mathscr{M}ocov$ $\{f^{-1}(K_i) : i \in J\}$ has a finite subcover say $\{f^{-1}(K_1), f^{-1}(K_2),$
$\ldots, f^{-1}(K_n)\}$. Since f is surjective $\{K_1, K_2, \ldots, K_n\}$ is a $\mathfrak{N}ocov$ of V, which is finite. Hence V is $\mathfrak{N}comp$.

Theorem 5.7. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}irr$ and a subset L of U is $\mathfrak{N}\mathscr{M}comp$ subset relative to U, then the image $f(L)$ is $\mathfrak{N}\mathscr{M}comp$ relative to V.

Proof. Let $\{K_i : i \in J\}$ be any collection of $\mathfrak{N}\mathscr{M}o$ subsets of V such that $f(L) \subset \bigcup\{K_i : i \in J\}$. Then $L \subset \bigcup\{f^{-1}(K_i) : i \in J\}$ where $\{f^{-1}(K_i) : i \in J\}$ is family of $\mathfrak{N}\mathscr{M}o$ sets in U. Since L is $\mathfrak{N}\mathscr{M}comp$ relative to U, the $\mathfrak{N}\mathscr{M}ocov$ $\{f^{-1}(K_i) : i \in J\}$ of U has a finite subcover say $\{f^{-1}(K_i) : i \in J_0\}$, where J_0 is a finite subset of J such that $L \subset \bigcup\{f^{-1}(K_i) : i \in J_0\}$. Therefore $f(L) \subset \bigcup\{K_i : i \in J_0\}$. Hence $f(L)$ is $\mathfrak{N}\mathscr{M}comp$ relative to V.

6. Nano $\mathscr{M}$ Connectedness

Definition 6.1. A $\mathfrak{N}ts$ $(U, \tau_R(P))$ is called Nano $\mathscr{M}$ connected (briefly, $\mathfrak{N}\mathscr{M}con$) if U is not the disjoint union of two nonempty $\mathfrak{N}\mathscr{M}o$ subsets.

Theorem 6.2. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}Cts$, surjection and U is $\mathfrak{N}\mathscr{M}con$ then V is $\mathfrak{N}con$.

Proof. Suppose V is not $\mathfrak{N}con$. Then $V = K \cup L$ where K and L are disjoint nonempty $\mathfrak{N}o$ sets in V. Since f is $\mathfrak{N}\mathscr{M}Cts$ surjection, $U = f^{-1}(K) \cup f^{-1}(L)$ where $f^{-1}(K)$ and $f^{-1}(L)$ are disjoint nonempty $\mathfrak{N}\mathscr{M}o$ subsets of U implies U is not $\mathfrak{N}\mathscr{M}con$ space. This is a contradiction to the hypothesis. Therefore V is $\mathfrak{N}con$.

Theorem 6.3. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathscr{M}irr$, surjection and U is $\mathfrak{N}\mathscr{M}con$ then V is also $\mathfrak{N}\mathscr{M}con$.

Proof. Suppose V is not $\mathfrak{N}\mathscr{M}con$. Then $V = K \cup L$ where K and L are disjoint nonempty $\mathfrak{N}\mathscr{M}o$ sets in V. Since f is $\mathfrak{N}\mathscr{M}irr$ surjection, $U = f^{-1}(K) \cup f^{-1}(L)$ where $f^{-1}(K)$ and $f^{-1}(L)$ are disjoint nonempty $\mathfrak{N}\mathscr{M}o$ subsets of U implies U is not $\mathfrak{N}\mathscr{M}con$ space. This is a contradiction to the hypothesis. Therefore V is $\mathfrak{N}\mathscr{M}con$.

Corollary 6.4. If $f\colon (U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ is bijective $\mathfrak{N}\mathscr{M}of$ and V is $\mathfrak{N}\mathscr{M}con$ then V is $\mathfrak{N}con$.

Theorem 6.5. For a $\mathfrak{N}ts$ $(U, \tau_R(P)$ the following are equivalent.

(i) U is $\mathfrak{N}\mathscr{M}con$.

(ii) The only subsets of U which are both $\mathfrak{N}\mathscr{M}o$ and $\mathfrak{N}\mathscr{M}c$ are the empty set $\emptyset$ and U.

(iii) Each $\mathfrak{N}\mathscr{M}Cts$ function of U into a discrete space V with atleast two points is a constant function.

Proof. (i) $\Rightarrow$ (ii): Suppose (i) holds and F is a proper subset of U which is both $\mathfrak{N}\mathscr{M}o$ and $\mathfrak{N}\mathscr{M}c$. Then $U-F$ is also both $\mathfrak{N}\mathscr{M}o$ and $\mathfrak{N}\mathscr{M}c$. Therefore $U = F\cup(U-F)$ is disjoint union of two nonempty $\mathfrak{N}\mathscr{M}o$ sets. This contradicts the fact that U is $\mathfrak{N}\mathscr{M}con$. Hence $F=\emptyset$ or U.

(ii) $\Rightarrow$ (i): Suppose (ii) holds. If possible U is not $\mathfrak{N}\mathscr{M}con$, then $U = K\cup L$ where K and L are disjoint non empty $\mathfrak{N}\mathscr{M}o$ sets in U. Since $K = U-L$, K is $\mathfrak{N}\mathscr{M}c$ set. But by assumption, $K=\emptyset$ or U which is contradiction. Hence (i) hold.

(ii) $\Rightarrow$ (iii): Let f be $\mathfrak{N}\mathscr{M}Cts$ function, where V is discrete space with atleast two points. Then $f^{-1}(\{m\})$ is both $\mathfrak{N}\mathscr{M}o$ and $\mathfrak{N}\mathscr{M}c$ for each $m\in V$ and $U=\bigcup\{f^{-1}(\{m\}\ ):m\in V\}$. By assumption, $f^{-1}(\{m\}) = U$ or $\emptyset$. If $f^{-1}(\{m\})=\emptyset$ for all $m\in V$, then f will not be function. Also there cannot exist more than one point $m\in V$ such that $f^{-1}(\{m\}) = U$ and $f^{-1}(m_1)=\emptyset$ where $m\neq m_1\in V$. This shows that f is constant function.

(iii) $\Rightarrow$ (ii): Let F be both $\mathfrak{N}\mathscr{M}o$ and $\mathfrak{N}\mathscr{M}c$ in U. Suppose $F\neq\emptyset$ and f be a $\mathfrak{N}\mathscr{M}Cts$ function defined by $f(F)=\{l\}$ and $f(U-F)=\{m\}$ for some distinct points l and m in V. By assumption f is constant function. Therefore $F=U$.

Conclusion

In this paper, we have studied many interesting notions on nano separation axioms via $\mathfrak{N}\mathscr{M}o$ sets and their respective $\mathfrak{N}\mathscr{M}$ compactness and $\mathfrak{N}\mathscr{M}$ connectedness.

References

[1] A. I. El-Maghrabi and M. A. Al-Juhani, *M-open sets in topological spaces,* Pioneer J. Math. Sci., **4** (2) (2011), 213–230.

[2] M. M. Khalaf and Ahmed Elmoasry, *Nano generalized closed sets and slightly Nano Separation Axioms*, **11** (1) (2015), 91-98.

[3] E. F. Lashin and T. Medhat, *Topological reduction of information systems*, Chaos, Solitons and Fractals, 25 (2015), 277-286.

[4] M. Lellis Thivagar and Carmel Richard, *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, 1 (2013), 31-37.

[5] A. Padma, M. Saraswathi, A. Vadivel and G. Saravanakumar, *New Notions of Nano $\mathscr{M}$-open Sets*, Submitted.

[6] A. Padma, M. Saraswathi and A. Vadivel, *More on generalizations of Nano $\mathscr{M}$-continuous functions*, Submitted.

[7] V. Pankajam and K. Kavitha, *δopen sets and δ nano continuity in δ nano topological space,* International Journal of Innovative Science and Research Technology, **2** (12) (2017), 110-118.

[8] Z. Pawlak, *Rough sets*, International Journal of Computer and Information Sciences, 11 (1982), 341-356.

[9] A. Revathy and I. Gnanambal, *Int. Jr. of Engineering, Contemporary Mathematics and Sciences*, 1 **(2)** (2015), 1-6.

[10] C. Richard, *Studies on nano topological spaces*, Ph. D. Thesis, Madurai Kamaraj University, India, (2013).

[11] S. B. Shalini, *A study on nano generalized β closed sets and its maps in nano topological spaces*, Ph.D Thesis, Nirmala College for Women(A), Coimbatore, (2018).

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