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Two level production inventory model with exponential demand and time dependent deterioration rate

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Abstract
In this paper, a production inventory model is developed by considering two different rates of production with exponential demand rate. It is assumed that rate of deterioration is linear function of time and shortages are not allowed. Production rates are different in two different phases of production. Likewise demand rate also changed. Optimal production inventory model is developed to determine the optimal production quantity and minimized the total cost, i.e. minimized the sum of production cost, setup cost, holding cost and deterioration cost. A numerical example for the verification of the problem is also given. Sensitivity analysis is also carried out for the different parameters.

Keywords
EPQ, time dependent deterioration rate, exponentially increasing demand, and different production.

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Contents

1. Introduction

Inventory level decrease with the demand rates and also, one of the factor decrease inventory level as well as profit which called deterioration and we cannot be ignored it. In general, deterioration is defined as spoilage, decay, damage, loss of utility, obsolescence etc. like fruits, fertilizer, food items, medicine. Some items have low rate of deterioration such as hardware, toys, glassware etc. Therefore, rate of deterioration is also different for different items. Shah and Jaiswal [1] discussed an inventory model by assuming constant rate of deterioration. Misra [2] investigated a deterministic model in which rate of replenishment is finite, shortage not allowed and rate of deterioration considered both variable and constant. Chaudhari and Chaudhari [3] developed an inventory model. They considered finite rate of replenishment and constant rate of deterioration. Balki and Benkherouf [4] developed a production lot size inventory model in their studied they considered arbitrary production rate and time dependent demand. Bhunia and Maiti [5] investigated deterministic inventory model for variables production. They also assumed production rate depends on either demand or on hand inventory. Chao and Chang [6] developed a production inventory model, they assumed that variables demand rate, production is proportional to demand without shortage and no deteriorating items. Wee and Wang [7] introduced a production inventory model by assuming finite rate of production, time dependent demand, deteriorating items and shortages. Sana et. al. [8] presented a production inventory model for deteriorating items over finite planning horizon with finite rate of production, time varying linear demand and constant deterioration rate. They also considered completely backlogged shortages. Yang and Wee [9] investigated an integrated multi-lot-size production inventory model in which rate of production and demand are constant with deteriorating items. Avikar et al. [10] presented a production inventory model for deteriorating items with exponentially increasing demand over a fixed time horizon, rate of production and deterioration both constant.

In the present study, we consider two different rates of production. Sivashankari and Panayappan [11] developed a production inventory model for two level production with deteriorative items and shortages. They considered rate of production, deterioration and demand was constant over time horizon. Islam et al. [12] presented a production model with constant production rate and assumed that market demands are exponentially decreasing. Patra and Maity [13] discussed a production inventory model in which variable production rate, deterioration and two type of demand rates for defective and non-defective items are considered.

In this paper, presented two level production both rate of production is constant, demand is the function of time which is increasing exponentially and rate of deterioration is also linear function of time.

2. Assumptions and Notations

Assumptions

(a) The demand rate is $a\exp(bt)$, where a and b are both constant such that $a>0$ and $b>0$.
(b) Two rates of production are considered.
(c) Production run only single product.
(d) The production rate is always greater than or equal to sum of the demand rate and deterioration rate.
(e) Rate of deterioration is linear function of time.
(f) Shortages are not allowed.

Notations

(1) P is initial production rate in units per unit time.
(2) I_1: On-hand inventory level at time T_1.
(3) I_2: On-hand inventory level at time T_2.
(4) I^* Optimal inventory.
(5) C_p production cost per unit time.
(6) C_h Holding cost per unit per year.
(7) C_0 Setup cost per set up.
(8) $\alpha+\beta t$ Rate of deterioration where $0<\alpha,\, \beta \ll 1$.
(9) T cycle time.
(10) T_i: unit time in period, $i=1,2,\ldots$
(11) T_c: Total cost.

3. Mathematical Formulation and it's Solutions

In mathematical formulation of production inventory model for two level production with above described assumptions and notations is depicted in Figure 1. Let the cycle starts at time $t=0$, and during the time interval $[0\ T_1]$, the production rate is P and the demand rate is ae^{bt} such that $P>ae^{bt}$ and the inventory level increases at the rate $P-ae^{bt}$. At time T_1 suppose that I_1 is maximum inventory level. During the time interval $[T_1\ T_2]$, the production rate is λP and demand rate is λae^{bt} where 'λ' is constant and $\lambda>0$ and hence, inventory level increases in this interval at the rate $\lambda[P-ae^{bt}]$. At time T_2 production stopped and suppose that I_2 is maximum inventory at that time. Due to demand and deterioration inventory level starts to decrease and after time T inventory becomes zero.

Let $I(t)$ be the inventory level at any time t where $0\le t\le T$. Then the model is governing by the following differential equations

$$\frac{dI}{dt}+(\alpha+\beta t)I=P-\ ae^{bt} \qquad 0\le t\le T_1 \qquad (3.1)$$
$$\frac{dI}{dt}+(\alpha+\beta t)I=\lambda(P-ae^{bt}) \qquad T_1\le t\le T_2 \qquad (3.2)$$
$$\frac{dI}{dt}+(\alpha+\beta t)I=-ae^{bt} \qquad T_2\le t\le T \qquad (3.3)$$

The boundary conditions of above Differential equations are given by

$$I(0)=0,\ I(T_1)=I_1,\ I(T_2)=I_2,\ I(T)=0 \qquad (3.4)$$

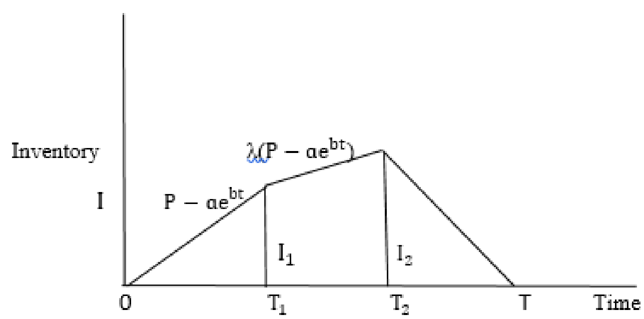

Figure 1

Using the boundary condition (3.4) and neglecting second and higher power of α and β, being α and β are very small then, the solution of equation (3.1), (3.2) and (3.3) are given by respectively

$$I=\frac{a}{b}\left(1-e^{bt}\right)+P\left(t-\frac{\alpha t^2}{2}-\frac{\beta t^3}{3}\right)-\frac{a\alpha}{b^2}(1+bt-e^{bt})$$
$$+\frac{a\beta}{b}[\frac{1}{b^2}-\frac{t^2}{2}+(\frac{t}{b}-\frac{1}{b^2})e^{bt}] \quad 0\le t\le T_1 \qquad (3.5)$$

$$I=\ \frac{a\lambda}{b}(1-e^{bt})+\ \lambda P\left(t-\frac{\alpha t^2}{2}-\frac{\beta t^3}{3}\right)$$
$$-\frac{a\lambda e^{bt}}{b}(1-\frac{\alpha+\beta t}{b}+\frac{\beta}{b^2})$$
$$-\left(\alpha t+\frac{\beta t^2}{2}\right)\left[\frac{a}{b}(1-e^{bT_1})+PT_1(1-\lambda)+\frac{a\lambda e^{bT_1}}{b}\right]$$
$$+\frac{a(1-e^{bT_1})}{b}+\frac{a\lambda e^{bT_1}}{b}\left(1-\frac{\alpha+\beta T_1}{b}+\frac{\beta}{b^2}\right)$$
$$+P(1-\lambda)\left(T_1-\frac{\alpha T_1^2}{2}-\frac{\beta T_1^3}{3}\right)-\frac{a\alpha(1+bT_1-e^{bT_1})}{b^2}$$

$$-\frac{a\beta}{b}\left(\frac{1}{b^2}-\frac{T_1^2}{2}\right)+\frac{a\beta}{b}\left(\frac{T_1}{b}-\frac{1}{b^2}\right)e^{bT_1}$$
$$+\left(\alpha T_1+\frac{\beta T_1^2}{2}\right)\left[\frac{a}{b}(1-e^{bT_1})+PT_1(1-\lambda)\right.$$
$$\left.+\frac{a\lambda e^{bT_1}}{b}\right] \quad T_1\le t\le T_2 \qquad (3.6)$$

$$I=\frac{a}{b}\left[\beta\left(\frac{t}{b}-\frac{1}{b^2}\right)+\frac{\alpha}{b}-1\right]e^{bt}-\frac{ae^{bT}}{b}(\alpha t+\frac{\beta t^2}{2})$$
$$+\frac{ae^{bT}}{b}[1+\alpha\left(T-\frac{1}{b}\right)+\beta(\frac{1}{b^2}+\frac{T^2}{2}-\frac{T}{b})]$$
$$T_2\le t\le T \qquad (3.7)$$

Total cost: The total cost is the sum of the production cost, setup cost, holding cost and deterioration cost.

Let us consider $T_1=\theta T_2$, where $\theta>0$.

Total Cost Function

$$T_c=\frac{aC_p}{bT}[(1-\lambda)e^{b\theta T_2}+(1+\lambda)e^{bT_2}-e^{bT}]+\frac{C_0}{T}$$
$$+\frac{C_h}{T}[\frac{a}{b^2}(b\theta T_2+1)\left(1-\frac{\alpha}{b}\right)$$
$$+\frac{P\theta^2T_2^2}{12}(6-2\alpha\theta T_2-\beta\theta^2T_2^2)-\frac{a\alpha\theta^2T_2^2}{b^2}$$
$$+\frac{a\beta}{6b^4}\{b\theta T_2(6-b^2\theta^2T_2^2)+12\}$$
$$+\frac{a}{b^4}\{\beta(b\theta T_2-2)+\alpha b-b^2\}e^{b\theta T_2}$$
$$+\frac{1}{6}\{\lambda P(3-\alpha)-3\alpha(1-\lambda)\left(P\theta T_2-\frac{ae^{b\theta T_2}}{b}\right)$$
$$+3a\alpha\}(1-\theta^2)T_2^2$$
$$+\frac{\beta}{6b}\{Pb+a-b(1-\lambda)\left(P\theta T_2-\frac{ae^{b\theta T_2}}{b}\right)\}$$
$$(1-\theta^3)T_2^3$$
$$+\frac{\beta P}{24}(1-\lambda)(1-\theta^4)T_2^4-\frac{\beta aT_2}{b^3}(e^{bT_2}-\theta e^{b\theta T_2})$$
$$+\frac{a}{b^4}\{\beta(1-\lambda)-b\lambda(b^2-\alpha)\}(e^{bT_2}-e^{b\theta T_2})$$
$$+\frac{a}{b}\{1+\alpha\left(T-\frac{1}{b}\right)+\beta\left(\frac{1}{b^2}+\frac{T^2}{2}-\frac{T}{b}\right)\}(T-T_2)$$
$$+\frac{a\beta}{b^2}\{\left(T-\frac{1}{b^2}\right)e^{bT}-(T_2-\frac{1}{b^2})e^{bT_2}\}$$
$$-\frac{ae^{bT}}{6b}\{3\alpha(T^2-T_2^2)+\beta(T^3-T_2^3)\}$$
$$+\frac{a}{b^3}(\alpha-b)(e^{bT}-e^{bT_2})]+\frac{P\theta^2T_2^2C_p}{6T}(3\alpha+2\beta\theta T_2)$$
$$+\frac{aC_p}{2b^3T}[2b\alpha(b\theta T_2+1)+\beta(b^2\theta^2T_2^2-2)$$
$$+2\{\beta(1-b\theta T_2)-b\alpha\}e^{b\theta T_2}]$$
$$+\frac{\lambda C_p}{T}\left[\frac{\alpha PT_2^2}{2}(1-\theta^2)+\frac{\beta PT_2^3}{3}(1-\theta^3)\right.$$
$$\left.-\frac{a}{b^3}(b\alpha-\beta)(e^{bT_2}-e^{b\theta T_2})-\frac{\beta aT_2}{b^2}(e^{bT_2}-\theta e^{b\theta T_2})\right]$$
$$+\frac{C_p}{T}\left[(1-\lambda)\left(P\theta T_2-\frac{ae^{b\theta T_2}}{b}\right)+\frac{a}{b}\right]$$
$$\left[\alpha T_2(1-\theta)+\frac{\beta T_2^2}{2}(1-\theta^2)\right]-\frac{aC_p}{2bT}\{2\alpha(T-T_2)$$
$$+\beta(T^2-T_2^2)\}+\frac{aC_p}{b^3T}[\{b\alpha+\beta(bT-1)\}e^{bT}$$
$$-\{b\alpha+\beta(bT_2-1)\}e^{bT_2}] \qquad (3.8)$$

The total cost function T_c involved two variables T and T_2. The values of T and T_2 for which the cost function T_c minimize is obtained by solving the simultaneous equations

$$\frac{\partial T_c}{\partial T}=0 \quad \text{and} \quad \frac{\partial T_c}{\partial T_2}=0 \qquad (3.9)$$

Provided that $\left(\frac{\partial^2 T_c}{\partial T^2}\right)>0$, $\left(\frac{\partial^2 T_c}{\partial T_2^2}\right)>0$ and

$$\left(\frac{\partial^2 T_c}{\partial T^2}\right)\left(\frac{\partial^2 T_c}{\partial T_2^2}\right)-\left(\frac{\partial^2 T_c}{\partial T\partial T_2}\right)^2>0 \qquad (3.10)$$

Equations (3.9) and (3.10) are solved numerically with the help of Mathematica.

4. Numerical Example

Let us consider an inventory model with the following data: $P=4000$, $a=600$, $b=0.3$, $\alpha=0.01$, $\beta=0.1$, $\theta=0.4$, $\lambda=2$, $C_0=80$, $C_p=40$, $C_h=2$.

On the basis of above cost parameters data, we obtain the optimal solution as: $T=5.831$, $I^*=20119$, $T_1=1.461$, $T_2=3.6524$, $I_1=4218.8$, $I_2=1177.8$, $T_c=31119.1$ Production cost $=22958.4$, Setup cost $=13.72$, Holding cost $=6706.1$, Deterioration cost $=1440.8$,

5. Sensitivity analysis

(1) Increasing of deterioration, cycle time T, optimal inventory I^*, production time (T_1 and T_2), maximum inventory I_1 and total cost T_c decreases but maximum inventory I_2 increases.

(2) Increasing of setup cost, cycle time T, optimal inventory I^*, production time (T_1 and T_2), maximum inventory I_1 and I_2 and total cost T_c remain unchanged.

(3) Increasing of holding cost, cycle time T, optimal inventory I^*, production time (T_1 and T_2), and maximum inventory I_1 decreases but total cost T_c and maximum inventory I_2 increases.

(4) Increasing of production cost, cycle time T, optimal inventory I^*, production time (T_1 and T_2), maximum inventory I_1 and total cost T_c increases but maximum inventory I_2 decreases.

Table 1. Variation in rate of deteriorating items with inventory and total cost

α	β	T	I^*	Production cost	Setup cost	Holding cost	Deteriorating cost	Total cost
0.01	0.1	5.831	20119	22958.4	13.72	6706.1	1440.8	31119.07
0.02	0.1	5.7695	19543	23015.5	13.87	6584.5	1234.8	30848.65
0.03	0.1	5.7085	18986	23075	14.01	6465.7	1028.7	30583.33
0.01	0.08	6.3963	26148	20529	12.51	7512.7	2852.9	30907.09
0.01	0.09	6.0937	22749	21905.5	13.13	7069.6	2069.4	31057.58

Table 2. Effect of demand and cost parameters on optimal values

α	β	T	I^*	T_1	T_2	I_1	I_2	T_c
0.01	0.1	5.831	20119	1.461	3.6524	4218.8	1177.8	31119.07
0.02	0.1	5.7695	19542.9	1.4412	3.6047	4145.8	1191	30848.65
0.03	0.1	5.7085	18985.6	1.423	3.5575	4074.4	1203.8	30583.33
0.01	0.08	6.3963	26148.3	1.6157	4.0393	4641.1	372.4	30907.09
0.01	0.09	6.0937	22749.4	1.5334	3.8334	4416.3	823.9	31057.58
C0	70	5.831	20119	1.461	3.6525	4218.9	1177.2	31115.6
	80	5.831	20119	1.461	3.6524	4218.8	1177.8	31119.07
	90	5.831	20119	1.461	3.6524	4218.8	1177.8	31119.07
Ch	1.5	5.89	20685.4	1.4732	3.6829	4243.2	992.1	29432.5
	2	5.831	20119	1.461	3.6524	4218.8	1177.8	31119.1
	2.5	5.7757	19600.3	1.45	3.6248	4196.5	1342	32786
Cp	35	5.799	19817.4	1.4545	3.6363	4205.8	1274	28066.4
	40	5.831	20119	1.4609	3.6524	4218.8	1177.8	31119.1
	45	5.8568	20365	1.466	3.6656	4229.4	1097.9	34166.8
b	0.1	5.6818	6017.2	0.4924	1.2311	1644.4	4698.3	43135.2
	0.2	5.9442	11709.8	1.3726	3.4319	4109.3	4580.3	32362.2
	0.3	5.831	20119	1.461	3.6524	4218.8	1177.8	31119.07
a	560	6.0059	20382.8	1.5502	3.8756	4475.4	621.2	32794.8
	600	5.831	20119	1.4609	3.6524	4218.8	1177.8	31119.1
	640	5.6816	19993.9	1.3828	3.457	3984.4	1488.7	29886.7
θ	0.4	5.831	20119	1.4609	3.6524	4218.8	1177.8	31119.1
	0.5	5.4688	16926.4	1.5863	3.1726	4456.6	2876.8	23915
	0.6	5.323	15770	1.7876	2.9794	4776.3	2817.1	21004.5
λ	2	5.831	20119	1.4609	3.6524	4218.8	1177.8	31119.1
	2.2	5.6439	18410.5	1.3503	3.3758	3986.3	2753.6	29112.7
	2.4	5.4978	17164.8	1.26	3.1499	3781.8	3957.6	27778.6

6. Conclusions

In this paper, we proposed two level production inventory models with exponential demand and time dependent deterioration rate. The objective is to determine optimal production policy and minimize the total inventory cost. The variation in production rate provides a way resulting consumer satisfaction and earning potential profit. The present model differs from the existing models, as two-level production and exponentially increasing demand are considered here. A numerical example is provided to demonstrate means practical usage. This model can be extended by taking more realistic assumptions such as shortage, other types of demand patterns etc.

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