Malaya Journal of Matematik, Vol. S, No. 1, 39-44, 2020

https://doi.org/10.26637/MJM0S20/0008

A discrete time perishable inventory model with positive lead time

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Abstract

This paper presents a perishable type inventory model in discrete time for the products with different life times. Arrival of customers constitutes a Bernoulli process. Replenishment rate, perishability of items and service time are geometrically distributed. The items are replenished according to (s,S) policy. Maximum storage of inventory is S. Considering positive lead time, we construct a multi-dimensional Markov chain to model the inventory-level process. When the level of inventory downs to s due to decay or service the items are replenished to the maximum level S. Using Matrix Analytic Method, we obtain steady state probability vector of the system. Various system performance measures in the steady state and expected total cost are obtained. Certain important numerical calculations were made and the results are illustrated graphically.

Keywords

Bernoulli process, perishable inventory, service-time, lead-time, Matrix Analytic Method.

AMS Subject Classification

60K25, 90B05 and 91B70

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Article History: Received **01** January **2020**; Accepted **12** February **2020**

Contents

1. Introduction

A perishable item is those item whose value likely to reduce over the time. Some examples include fruits, baked food, fashion items, etc. Many factors which are reasons for decay or damage of perishable product include, low product life, change in atmospheric condition, changes in fashion trend, etc. Since perishability affect inventory level, service, reorder level and cost of item, such inventory management is more challenging. Academicians as well as administrators are equally concerned about perishable inventory model because of its wide application.

A model studied by Whitin [1] in 1957 on deterioration of fashion item is considered as the first work on perishability. A recent study by Feng et al. [2] had taken into account many new aspects of such problem including items displayed, freshness of items, price dependency etc. Effect of discount on inventory models was first studied by Taleizadeh, et al. [3] by considering customer's decision as the factor. Some possible cases were investigated in that paper and suggestions were given for finding the closed form solution when objective function involves integer operators. Nowadays, in addition to rate of perishability and constant demand, various factors such as level dependent inventory, time dependent demand, price dependent demand, etc. were taken into account. Some assumptions on perishability rate found in literature include, perishable rate as a constant, perishable rate as a linear function, perishable rate as a Weibull distribution, etc.

Lian and Liu [4] analysed a perishable inventory model by incorporating life time of the items. Considering zero lead time in an (s,S) continuous inventory model, they obtained a closed form solution to the steady state probability distribution using Markov renewal approach. Nandakumar and Morton [5] applied a heuristics method for the perishability model constructed by assuming ordering cost, holding cost and penalty cost to be linear and i.i.d. demand. In the classic newsboy model frame work with periodic review, they used the method of developing heuristics near myopic bounds. Using numerical experiments, they showed that the results obtained coincide with the optimal values.

Kalpakam and Arivargnan [6] developed an (s,S) continuous review inventory model with perishability. Assuming Poisson arrivals, exponential perishability and zero lead time, the expected successive reorder time and steady state probability distribution of stock level were derived. Liu [7] considered a continuous review (s,S) inventory system with random demands and random failures by assuming instantaneous replenishment and allowing backlogs. Stationary probability distribution and other important system performance measures were derived explicitly in an alternative approach by assuming demand to be Poisson and life time follows exponential distribution. Pal [8] developed a continuos time Markovian inventory model by assuming that inter demand time follows a general density function. Assuming zero lead time, they obtained a solution which minimizes the expected total cost per unit time.

A model in the continuous review system with damages in the items of inventory due to decay and disaster were considered by Krishnamoorthy and Varghese [9]. By assuming Poisson demands, exponential inter-disaster times and exponential lifetimes they derived the steady state probabilities. Kalpakam and Sapna [10] studied a perishable inventory system with Poisson demands and positive lead time. Assuming constant decay rate and exponential life times, they applied Markovian renewal technique to get system characteristics. Sivakumar [11] studied a perishable inventory system with retrial demands from finite homogenous sources. They obtained expected number of demands in the orbit and joint probability distribution of inventory level. Kalpakam and Shanti [12] developed a lost sales $(S-1,S)$ perishable inventory model with variable ordering quantity.

In this model, process of arrival is Bernoulli. Lead time, service time and decay time are geometric. Arriving customers cannot enter the system during the stock out period. We adopt (s,S) policy and place replenishment order when the inventory level reaches the reorder level s due to decay or service completion. The condition for the stability of the system and system characteristics are obtained. Matrix Analytic Method is used to analyse the inventory system and to develop algorithmically tractable solutions described in Neuts [13] and Latouche and Ramaswamy [14]. Illustrations based on numerical experiments are presented for different parameter values. The paper is organized as follows. Section 2 deals with the assumptions and notations used for modelling and analysis. In section 3, the stability and steady state analysis is done and in session 4, some relevant performance measures are defined. In section 5, the expected cost function of the model is modelled. In section 6, certain numerical results are presented and its graphical illustrations are given in section 7. Finally in section 8, concluding remarks and future works are discussed.

2. Mathematical Modelling and Analysis

The following are the assumptions and notations used in this model. **Assumptions**

1. Inter-arrival times are geometrically distributed with parameter a.
2. Service times are geometricaly distributed with parameter b.
3. Lead times are geometrically distributed with parameter c.
4. Life times of items are geometrically distributed with parameter $l_j^{(i)}$.

In practice, decay of inventory is either one by one or all at a time (common life time). Here, life time is assumed to be geometric with parameter $l_j^{(i)}$, i.e. out of i items in the stock, any number of items $0 \le j \le i$, can be perished in an epoch.

Notations

$X(n)$: Number of customers in queue at an epoch n.

$Y(n)$: Inventory level at the epoch n.

$\mathbf{e}$: $(1,1,1,...,1)'$, column vector of 1's of size $(S+1)$.

Then $\{(X(n),Y(n)); n = 0,1,2,3,..\}$ is a Quasi-Birth-Death Process on the state space

$$E = \{(i,j); i \ge 0, 0 \le j \le S\}.$$

Now, the transition probability matrix of the process is

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & \dots \end{array} \\ \begin{pmatrix} C_1 & C_0 & & & \\ A_2 & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & A_2 & A_1 & A_0 \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \end{array}$$

where C_0, C_1, A_0, A_1 and A_2 are block matrices of order $(S+1)\times(S+1)$, given by

$$[C_0]_{ij} = \begin{cases} ac, & j = S+1;\ i = 2,3,\dots,s+1 \\ a\bar{c}l_{i-j}^{(i-1)}, & 2 \le j \le i;\ i = 2,3,\dots,s+1 \\ al_{i-j}^{(i-1)}, & 2 \le j \le i;\ i = s+2,s+3,\dots,S+1 \end{cases}$$

$$[C_1]_{ij} = \begin{cases} \bar{c}, & j=1; i=1 \\ c, & j=S+1; i=1 \\ \bar{a}c, & j=S+1; i=2,3,\ldots,s+1 \\ \bar{c}l_{i-1}^{(i-1)}, & j=1; i=2,3,\ldots,s+1 \\ \bar{a}\bar{c}l_{i-j}^{(i-1)}, & 2\le j\le i; i=2,3,\ldots,s+1 \\ al_{i-1}^{(i-1)}, & j=1;\ i=s+2,s+3,\ldots,S+1 \\ \bar{a}l_{i-j}^{(i-1)}, & 2\le j\le i;\ i=s+2,s+3,\ldots,S+1 \end{cases}$$

$$[A_0]_{ij} = \begin{cases} a\bar{b}c, & j=S+1;\ i=2,3,\ldots,s+1 \\ a\bar{b}cl_{i-j}^{(i-1)}, & 2\le j\le i;\ i=2,3,\ldots,s+1 \\ a\bar{b}l_{i-j}^{(i-1)}, & 2\le j\le i;\ i=s+2,s+3,\ldots,S+1 \end{cases}$$

$$[A_1]_{ij} = \begin{cases} \bar{c}, & j=1; i=1 \\ c, & j=S+1; i=1 \\ \bar{a}\bar{b}c, & j=S+1; i=2,3,\ldots,s+1 \\ abc, & j=S; i=2,3,\ldots,s+1 \\ ab\bar{c}l_{i-2}^{(i-1)}+\bar{c}l_{i-1}^{(i-1)}, & j=1; i=2,3,\ldots,s+1 \\ \bar{a}\bar{b}\bar{c}l_{0}^{(i-1)}, & j=i; i=2,3,\ldots,s+1 \\ \bar{a}\bar{b}l_{0}^{(i-1)}, & j=i; i=s+2,s+3,\ldots,S+1 \\ ab\bar{c}l_{i-j-1}^{(i-1)}+\bar{a}\bar{b}\bar{c}l_{i-j}^{(i-1)}, & 2\le j\le i-1; i=3,4,\ldots,s+1 \\ abl_{i-j-1}^{(i-1)}+\bar{a}bl_{i-j}^{(i-1)}, & 2\le j\le i-1; \\ & i=s+2,s+3,\ldots,S+1 \end{cases}$$

$$[A_2]_{ij} = \begin{cases} \bar{a}bc, & j=S; i=2,3,\ldots,s+1 \\ \bar{a}b\bar{c}l_{i-j-1}^{(i-1)}, & 1\le j\le i-1; i=2,3,\ldots,s+1 \\ \bar{a}bl_{i-j-1}^{(i-1)}, & 1\le j\le i-1; i=s+2,s+3,\ldots,S+1 \end{cases}$$

3. Stability and Steady State Analysis

Since the arriving customers are not allowed to enter the system when the stock in the inventory is empty, the inventory level will not affect the length of the queueing system. Hence the stability of the considered queueing inventory model is same as that of ordinary discrete time Geo/Geo/1 queue with inter arrival time and service time are geometrically distributed with parameters say a and b. Then system is sable if $a<b$.

Assume that the QBD is aperiodic and positive recurrent. Denote by $\mathbf{x}$ its stationary probability vector. It is the unique solution of the system $\mathbf{x}P=\mathbf{x}$ and $\mathbf{xe}=1$, where $\mathbf{e}$ is a column vector of ones of appropriate order.
Let $\mathbf{x}$ be partitioned by levels as $\mathbf{x}=(\mathbf{x_0},\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}\ldots,)$. Then $\mathbf{x}_i$ has the matrix geometric form $\mathbf{x}_i=\mathbf{x}_1R^{i-1}, i\ge 2$. Where R is the minimal non negative solution of the matrix quadratic equation

$$R^2A_2+RA_1+A_0=R$$

. Where $\mathbf{x}_0$ and $\mathbf{x}_1$ are obtained by solving the equations

$$\mathbf{x}_0(C_0-I)+\mathbf{x}_1A_2=0$$

and

$$\mathbf{x}_0C_1+\mathbf{x}_1(A_1+RA_2-I)=0$$

with the normalizing condition

$$\mathbf{x_0e}+\mathbf{x}_1(I-R)^{-1}\mathbf{e}=1$$

where $\mathbf{e}$ is a column vector of 1's of size $(S+1)$.

4. System Performance Measures

Let the components of $\mathbf{x}=(\mathbf{x_0},\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}\ldots,)$ be $\mathbf{x_i}=(x_{i,0},x_{i,1},x_{i,2}\ldots,x_{i,S}),(i\ge 0)$. The relevant Performance measures of the system under steady state are given below

1. Expected inventory level, EI, is given by
$$EI=\sum_{i=0}^{\infty}\sum_{j=1}^{S} jx_{i,j}$$
2. Expected perishable rate, EP, is given by
$$EP=\sum_{j=1}^{S}\sum_{k=1}^{j}k\sum_{i=0}^{\infty}x_{i,j}l_k^{(j)}$$
3. Expected number of customers in the system, EC, is given by
$$EC=\sum_{i=1}^{\infty}i\mathbf{x_i e}$$
4. Expected reorder rate, ER, is given by
$$ER=\sum_{i=1}^{\infty}\sum_{j=s+1}^{S}x_{ij}\left[bl_{j-(s+1)}^{(j)}+\bar{b}l_{j-s}^{(j)}\right]$$
5. Expected rate of departure after completing the service, ED, is given by
$$ED=b\sum_{i=1}^{\infty}\sum_{j=1}^{S}x_{i,j}$$
6. Expected replenishment rate, ERR, is given by
$$ERR==c\sum_{j=0}^{s}\sum_{i=0}^{\infty}x_{i,j}.$$

5. Cost Analysis

Define the expected total cost of the system per unit time as

$$ETC=c_o(ER)+c_1\sum_{j=0}^{s}(S-j)x_jc+c_2(EI)$$
$$+c_3(EW)+c_4(EL)+c_5(EP)$$

where,

c_0 : The fixed ordering cost

c_1 : Procurement cost/unit

c_2 : Holding cost of inventory/unit/unit time

c_3 : Holding cost of customers/unit/unit time

c_4 : Cost due to loss of customers/unit/unt time

c_5 : Disposal cost/unit perished

6. Numerical Illustration

Tables below represents the variations of various performance measures with respect to the variations of parameters a, b and c respectively. Table 1 show that as a increases, expected inventory level, expected reorder rate and expected replenishment rate are decrease but expected number of customers, expected departure rate and expected perishability rate are increase. Table 2 show that as b increases, expected replenishment rate and expected reorder rate increases whereas expected inventory decreases. From table 3 we notice that as c increases, the expected inventory level and expected replenishment rate increases and at the same time expected inventory level decreases.

a	EC	EI	ER	ERR	ED	EP	ETC
l = .18							
0.38	1.3859	2.2419	0.0843	0.1002	0.7622	1.0466	12.941
0.39	1.4868	2.1521	0.0808	0.0970	0.0947	1.2878	12.82
0.40	1.5999	2.0583	0.0772	0.0935	0.8799	1.15	12.717
0.41	1.7276	1.9604	0.0735	0.0898	0.9502	1.2139	12.638
0.42	1.8733	1.8582	0.0696	0.0856	1.0303	1.2884	12.59
0.43	2.0413	1.7516	0.0655	0.0815	1.1227	1.3762	12.585
0.44	2.2373	1.6404	0.0613	0.0769	1.2305	1.4805	12.639
0.45	2.4694	1.5246	0.0569	0.0720	1.3581	1.6062	12.772
0.46	2.7481	1.4044	0.0524	0.0668	1.5114	1.7594	13.012
l = .20							
0.38	1.3859	2.1988	0.0860	0.1046	0.7622	1.1405	13.264
0.39	1.4868	2.1108	0.0824	0.1012	0.8177	1.193	13.159
0.40	1.5999	2.0189	0.0787	0.0975	0.8799	1.253	13.073
0.41	1.7276	1.923	0.0749	0.0936	0.9502	1.323	13.015
0.42	1.8733	1.8228	0.0709	0.0894	1.0303	1.4043	12.991
0.43	2.0413	1.7183	0.0668	0.0848	1.1227	1.5001	13.015
0.44	2.2373	1.6093	0.0625	0.0800	1.2305	1.6139	13.102
0.45	2.4694	1.4958	0.0580	0.0749	1.3581	1.7509	13.275
0.46	2.7481	1.378	0.0533	0.0695	1.5114	1.9181	13.565
l = .22							
0.38	1.3859	2.1577	0.0871	0.1088	0.7622	1.2311	13.556
0.39	1.4868	2.0715	0.0835	0.1052	0.8177	1.2878	13.467
0.40	1.5999	1.9814	0.0798	0.1014	0.8799	1.353	13.4
0.41	1.7276	1.8874	0.0759	0.0972	0.9502	1.4284	13.362
0.42	1.8733	1.7892	0.0718	0.0928	1.0303	1.5162	13.363
0.43	2.0413	1.6867	0.0676	0.0881	1.1227	1.6197	13.416
0.44	2.2373	1.5798	0.0633	0.0830	1.2305	1.7427	13.537
0.45	2.4694	1.4685	0.0587	0.0777	1.3581	1.8908	13.75
0.46	2.7481	1.3528	0.0540	0.0720	1.5114	2.0714	14.09

Table 1. Effect of b, c and l on the model. Where $s = 2, S = 7, b = 0.55, c = 0.4, c_0 = 50, c_1 = 1, c_2 = 2, c_3 = 3, c_4 = 2, c_5 = 2$.

b	EC	EI	ER	ERR	ED	EP	ETC
l = .18							
0.46	2.0663	5.2362	0.0370	0.0662	0.9505	4.184	26.809
0.47	1.8949	5.5248	0.0392	0.0698	0.8906	3.9023	26.656
0.48	1.7496	5.7944	0.0412	0.0733	0.8398	3.6666	26.603
0.49	1.6248	6.0463	0.0431	0.0765	0.7961	3.4671	26.62
0.50	1.5166	6.2819	0.0449	0.0794	0.7583	3.2966	26.689
0.51	1.4218	6.5024	0.0466	0.0822	0.7251	3.1495	26.794
0.52	1.3382	6.709	0.0481	0.0849	0.6959	3.0217	26.926
0.53	1.2639	6.9027	0.0497	0.0873	0.6699	2.9099	27.075
0.54	1.1974	7.0845	0.0511	0.0896	0.6466	2.8114	27.238
l = .20							
0.46	2.0663	5.1393	0.0372	0.0704	0.9505	4.5629	27.789
0.47	1.8949	5.4226	0.0393	0.0743	0.8906	4.2557	27.547
0.48	1.7496	5.6872	0.0413	0.0779	0.8398	3.9986	27.416
0.49	1.6248	5.9345	0.0432	0.0813	0.7961	3.7811	27.367
0.50	1.5166	6.1657	0.0450	0.0845	0.7583	3.5951	27.378
0.51	1.4218	6.3821	0.0466	0.0875	0.7251	3.4348	27.432
0.52	1.3382	6.5849	0.0482	0.0902	0.6959	3.2954	27.519
0.53	1.2639	6.775	0.0497	0.0928	0.6699	3.1734	27.628
0.54	1.1974	6.9534	0.0511	0.0953	0.6466	3.066	27.754
l = .22							
0.46	2.0663	5.0464	0.0371	0.0744	0.9505	4.9284	28.721
0.47	1.8949	5.3245	0.0392	0.0785	0.8906	4.5966	28.391
0.48	1.7496	5.5844	0.0412	0.0823	0.8398	4.3189	28.185
0.49	1.6248	5.8272	0.0431	0.0859	0.7961	4.0839	28.071
0.50	1.5166	6.0542	0.0448	0.0893	0.7583	3.8831	28.025
0.51	1.4218	6.2667	0.0464	0.0924	0.7251	3.7099	28.029
0.52	1.3382	6.4658	0.0479	0.0954	0.6959	3.5594	28.071
0.53	1.2639	6.6524	0.0494	0.0981	0.6699	3.4276	28.141
0.54	1.1974	6.8277	0.0508	0.1007	0.6466	3.3116	28.231

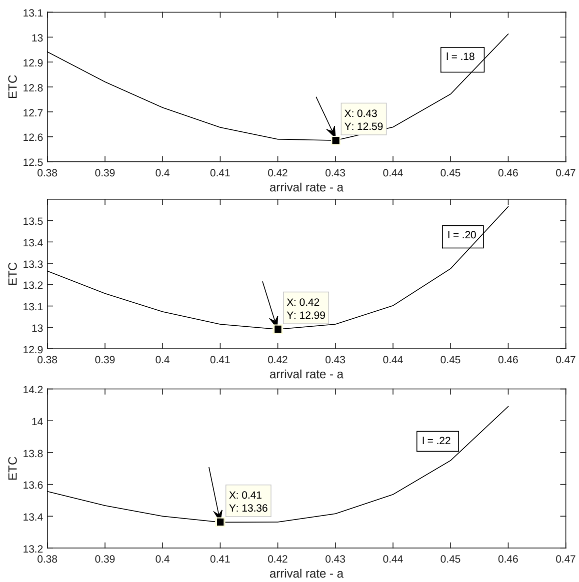
Table 2. Effect of a, c and l on the model. Where $s = 6, S = 20, a = 0.35, c = 0.4, c_0 = 80, c_1 = 1, c_2 = 2, c_3 = 4, c_4 = 5, c_5 = 2$.

c	EC	EI	ER	ERR	ED	EP	ETC
l = .18							
0.10	3.6569	1.485	0.0163	0.0227	1.0971	2.8515	22.204
0.11	3.6569	1.5705	0.0172	0.0241	1.0971	3.016	22.114
0.12	3.6568	1.6496	0.0181	0.0254	1.0971	3.1679	22.047
0.13	3.6568	1.7228	0.0189	0.0266	1.097	3.3085	22.001
0.14	3.6568	1.7908	0.0197	0.0277	1.097	3.4391	21.973
0.15	3.6567	1.854	0.0204	0.0288	1.097	3.5606	21.959
0.16	3.6567	1.9129	0.0211	0.0298	1.097	3.6739	21.958
0.17	3.6567	1.968	0.0217	0.0307	1.097	3.7798	21.968
0.18	3.6567	2.0195	0.0223	0.0316	1.097	3.8789	21.987
l = .20							
0.10	3.6569	1.4113	0.0161	0.0234	1.0971	3.0113	23.153
0.11	3.6569	1.4959	0.0171	0.0249	1.0971	3.192	23.101
0.12	3.6568	1.5744	0.0180	0.0263	1.0971	3.3596	23.069
0.13	3.6568	1.6475	0.0189	0.0276	1.097	3.5156	23.053
0.14	3.6568	1.7156	0.0197	0.0289	1.097	3.6609	23.052
0.15	3.6567	1.7791	0.0205	0.0300	1.097	3.7967	23.062
0.16	3.6567	1.8386	0.0211	0.0311	1.097	3.9237	23.084
0.17	3.6567	1.8944	0.0218	0.0321	1.097	4.0428	23.115
0.18	3.6567	1.9468	0.0224	0.0331	1.097	4.1547	23.153
l = .22							
0.10	3.6569	1.3448	0.0159	0.0241	1.0971	3.1566	24.035
0.11	3.6569	1.4283	0.0169	0.0257	1.0971	3.3526	24.023
0.12	3.6568	1.5061	0.0179	0.0272	1.0971	3.5353	24.026
0.13	3.6568	1.5787	0.0188	0.0286	1.097	3.7059	24.042
0.14	3.6568	1.6467	0.0196	0.0299	1.097	3.8655	24.069
0.15	3.6567	1.7104	0.0204	0.0311	1.097	4.0151	24.107
0.16	3.6567	1.7702	0.0211	0.0323	1.097	4.1555	24.153
0.17	3.6567	1.8264	0.0218	0.0334	1.097	4.2875	24.206
0.18	3.6567	1.8793	0.0225	0.0345	1.097	4.4119	24.265

Table 3. Effect of a, b and l on the model. Where $s = 4, S = 15, a = 0.25, b = 0.3, c_0 = 10, c_1 = 2, c_2 = 2, c_3 = 7, c_4 = 3, c_5 = 1$.

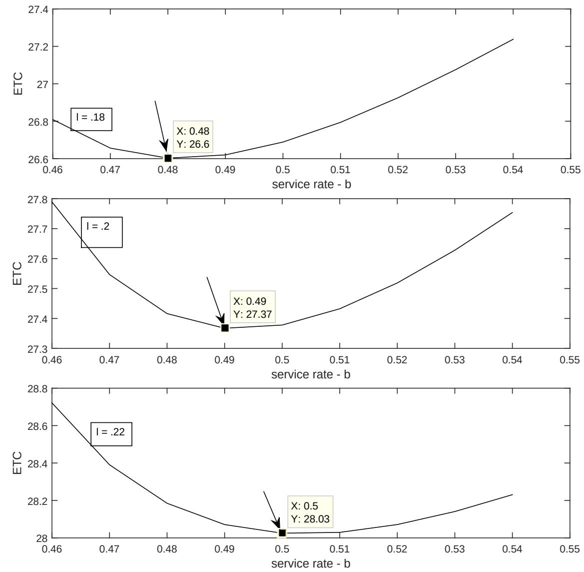
7. Graphical Illustrations

One can plot the graphs of ETC by varying the parameters a, b and c by keeping other parameters constant. These graphs indicate that on long run, the company will have expected minimum total cost at the indicated points. From figure 1, minimum values of a and ETC are $(0.43, 12.58), (0.42, 12.99)$ and $(0.41, 13.36)$ for values $0.18, 0.20$ and 0.22 of l respectively. From figure 2, minimum values of b and ETC are $(0.48, 26.6), (0.49, 27.37)$ and $(0.5, 28.03)$ for values 0.18, 0.20 and 0.22 of l respectively. From figure 3, minimum values of c and ETC are $(0.16, 21.96), (0.14, 23.05)$ and $(0.11, 24.02)$ for values $0.18, 0.20$ and 0.22 of l respectively.



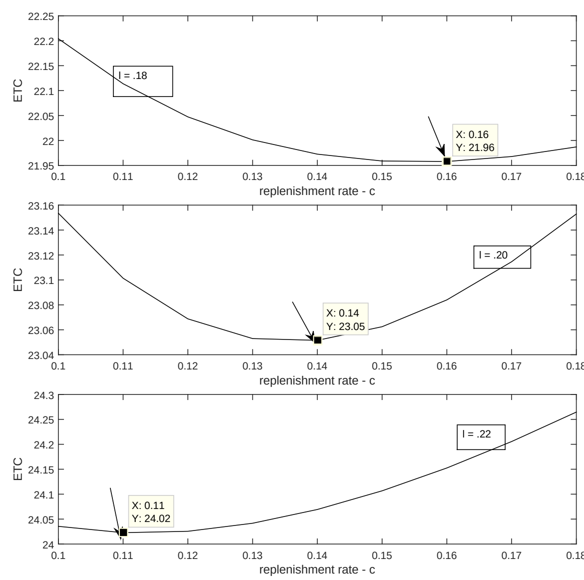
$S = 7; s = 2; C_0 = 50; c_1 = 1; c_2 = 2; c_3 = 3; C_4 = 2; c_5 = 2; b = 0.55; c = 0.4.$

Figure 1. a vs ETC



$S = 20; s = 6; C_0 = 80; c_1 = 1; c_2 = 2; c_3 = 4; C_4 = 5; c_5 = 2; a = 0.35; c = 0.4.$

Figure 2. b vs ETC



$S = 15; s = 4; C_0 = 10; c_1 = 2; c_2 = 2; c_3 = 7; C_4 = 3; c_5 = 1; a = 0.25; b = 0.3.$

Figure 3. c vs ETC

8. Concluding Remarks

In this article, we studied a discrete (s, S) perishable inventory system with positive service time and lead time. The primary demand constitutes a geometric process. Service time as well as perishability follow geometric distribution. Matrix Analytic Method is used to analyse the model. Some Important system performance measures are derived. The expression for expected total cost is also obtained. Future work includes the study of new model with backlog when the level of inventory is zero.

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