Study of heat and mass transport in temperature-dependent-viscous fluid under gravity modulation

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Abstract

In this paper, we study the thermosolutal convection in a horizontal temperature dependent viscous fluid layer. The considered gravity field consists of two parts: a steady part and a time-dependent periodic part that oscillates with time. The time periodic gravity modulation, can be realized by vertically oscillating the fluid layer. A weak non-linear stability analysis has been performed by using power series expansion in terms of the amplitude of gravity modulation, which is assumed to be small. The Nusselt and Sherwood numbers have been obtained in terms of the amplitude of convection which is governed by the non-autonomous Ginzburg-Landau equation derived for the stationary mode of convection. Effects of various parameters such as frequency and amplitude of modulation, Prandtl number, diffusivity ratio and solute Rayleigh number, have been analyzed on heat and mass transfer. It is found that heat and mass transport can be controlled by suitably adjusting the external parameters of the system. It is also found that the thermo-rheological parameter is to destabilize the system.

Keywords: Heat and mass transfer, Ginzburg-Landau equation, Temperature-dependant viscosity, Gravity modulation.

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1 Introduction

Double diffusive convection is an important fluid dynamics phenomenon. It is a type of instability that occurs in a fluid that possesses two opposing density altering components with differing molecular diffusivity, such as heat and salt or any two solute concentrations. The marked difference between single and double diffusive systems lies in the verity that in double diffusive systems convection can occur even when the system is hydrostatically stable if the diffusivities of the two diffusing fields are widely different. The study of the double diffusive convection has received much attention over the years due to its numerous fundamental and industrial applications. Some examples of double diffusive convection can be found in oceanography Stommel [1], lakes and under ground water, atmospheric pollution, chemical processes, laboratory experiments, modeling of solar ponds, astrophysics, geophysics, geology and engineering Chen and Johnson [2], magma chambers and sparks, formation of microstructure during the cooling of molten metals, fluid flows around shrouded heat-dissipation fins, migration of moisture through air contained in fibrous insulations, grain storage system, the dispersion of contaminants through water saturated soil, crystal growth, solidification of binary mixtures, and the underground disposal of nuclear wastes. The early work on this problem is summarized in several reviews Turner [3], Turner and Chen [4], Huppert and Turner [5], Bhadauria [6].

Stommel et al. [1] were the first to notice some properties of double diffusive convection with the discovery of the phenomenon of the salt fountain, which occurs when hot salty water lies above cold fresh
water. Such a system was later analyzed by Stern [7], who noted the general properties of the motion now commonly known as salt fingers. The situation with reversed gradients has been studied by Veronis [8], and stability criteria for horizontal boundaries of various kinds have been presented by Nield [9] by means of a linear stability analysis. Lortz [10] studied the effect of magnetic field on double-diffusive convection. His object was to clarify some of the mathematical aspects of stability criterion (Malkus and Veronis [11]) but, his analysis is silent about the detailed study of stability analysis. Considering linear gradients, Baines and Gill [12] investigated the thermohaline convection in a fluid layer confined between two horizontal boundaries, which are dynamically free and conducting to both heat and salt. Chen [13] considered a two-dimensional problem of a linearly stratified salt solution contained between two infinite vertical plates, and studied the onset of cellular convection due to a lateral temperature gradient. Proctor [14] studied the thermohaline convection in a horizontal fluid layer using rigid-rigid and free-free boundaries. Double diffusive convection in an inclined plane was investigated by Thangam et al. [15] for rigid-rigid boundaries. Later on many other investigators studied this problem of double diffusive convection under various physical and boundary conditions. Sodha and Kumar [16] studied the stability of double diffusive convection in solar ponds with non-constant temperature and salinity gradients. Using linear stability analysis, Saunders et al. [17] studied the effect of gravity modulation on thermosolutal convection in an infinite layer of fluid using free-free boundaries. Gobin and Bennacer [18] investigated the problem of thermohaline convection in a vertical layer of a binary fluid, and studied the onset of convection. Sezai and Mohamad [19] have performed a three-dimensional numerical study to investigate double diffusive, natural convection in a cubic enclosure subject to opposing and horizontal gradients of heat and solute imposed along the two vertical side walls. Ryskin et al. [20] have studied thermodynamical convection in ferrofluids. Bajaj [21] considered the thermosolutal magnetic convection in an infinite layer of ferrofluid heated from below, in the presence of vertical magnetic field and the vertical acceleration is modulated by two-frequency modulation. Bhadauria [22] analyzed this problem by considering rigid boundaries under temperature modulation. Starchenko [23] discussed double diffusion magneto-convection for Earths type planets. Siddheshwar et al. [24] performed a local non-linear stability analysis of Rayleigh–Bénard magneto-convective using Ginzburg-Landau equation. Magnetohydrodynamic natural convection boundary layer flow of an electrically conducting and viscous incompressible fluid along a heated vertical flat plate with uniform heat and mass flux in the presence of strong cross magnetic field has been investigated by Sadia [25].

The classical Rayleigh-Bénard convection due to bottom heating is well known and highly explored phenomenon given by Chandrasekhar [25], Drazin and Reid [26]. Gershuni and Zhukhovitskii [27] and Gresho and Sani [28] were the first to study the effect of gravity modulation in a fluid layer. Biringen and Peltier [29] investigated, numerically, the non-linear three dimensional Rayleigh-Bénard problem under gravity modulation, and confirmed the result of Gresho and Sani [28]. Wadih and Roux [30] presented a study on instability of the convection in an infinitely long cylinder with gravity modulation oscillating along the vertical axis. Saunders et al. [17] have discussed the effect of gravity modulation on thermosolutal convection in an infinite layer of fluid. Clever et al. [31] performed a detailed non-linear analysis of Rayleigh–Bénard convection under g-jitter and presented the stability limits to a much wider region of parameter space. Chen and Chen [32] have studied the effect of gravity modulation on the stability of convection in a vertical slot. They have examined the stability for fluids of different Prandtl numbers. Rogers et al. [33] have observed superlattice patterns in vertically oscillated Rayleigh Bénard convection. Rogers et al. [34], Bhadauria et al. [35] showed that the gravitational modulation, which can be realized by vertically oscillating a horizontal liquid layer, acts on the entire volume of liquid and may have a stabilizing or destabilizing effect depending on the amplitude and frequency of the forcing. Shu et al. [36] examined the effects of modulation of gravity and thermal gradients on natural convection in a cavity, numerically as well as experimentally. They found that for low Prandtl number fluids, modulations in gravity and temperature produce the same flow field both in structure and in magnitude. Boulal et al. [37] focused attention on the influence of a quasi-periodic gravitational modulation on the convective instability threshold. They predicted that the threshold of convection corresponds precisely to quasi-periodic solutions. Bhadauria et al. [38] studied thermally or gravity modulated non-linear stability problem in a rotating viscous fluid layer, using Ginzburg-Landau equation for stationary mode of convection. Bhadauria et al. [39] studied internal heating effects on weak non-linear Rayleigh–Bénard convection under gravity modulated, using Ginzburg-Landau equation for stationary mode of convection.

Most of the above studies considered only constant viscosity, however, in nature and in engineering
problems of convective flow, viscosity of many fluids varies with temperature. Therefore, the results drawn from the flow of fluids with constant viscosity are not applicable for the fluid that flows with temperature dependent viscosity, particularly at high temperature. The fluids that flow with variable viscosity are useful in chemical, bio-chemical and process industries as well as in physics of fluid flows, wherein the flow of fluids is governed by different temperatures. Therefore, objective of our study is to investigate the effect of gravity modulation on double diffusive convection in a horizontal temperature dependant fluid layer.

2 Governing Equations

We consider an infinite horizontal layer of temperature dependent viscous fluid mixture subjected to a vertical gravity field, confined between two free-free boundaries at \( z = 0 \) and \( z = d \). To maintain a constant temperature difference \( \Delta T \) and a constant solutal difference \( \Delta S \), across the layer, the layer is heated and salted from below. A Cartesian frame of reference is chosen with origin in the lower boundary and the \( z \)-axis vertically which is given in Fig.1. given bellow.

![Fig.1 : Physical configuration of the problem](image)

The fluid layer is considered to be Boussinesq, and thus the basic governing equations are

\[
\nabla \cdot \vec{q} = 0, \quad (2.1)
\]

\[
\rho_0 \left( \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right) = -\nabla p + \rho \vec{g}(t) \vec{k} + \mu(T) \nabla^2 \vec{q}, \quad (2.2)
\]

\[
\gamma \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa_T \nabla^2 T, \quad (2.3)
\]

\[
\frac{\partial S}{\partial t} + (\vec{q} \cdot \nabla) S = \kappa_S \nabla^2 S, \quad (2.4)
\]

\[
\vec{g}(t) = g_0 \left[ 1 + \epsilon^2 \delta \cos(\omega t) \right] \vec{k}, \quad (2.5)
\]

\[
\rho = \rho_0 \left[ 1 - \beta_T (T - T_0) + \beta_S (S - S_0) \right], \quad (2.6)
\]

\[
\mu(T) = \frac{\mu_0}{1 + \epsilon^2 \delta_0 (T - T_0)}, \quad (2.7)
\]

where \( \kappa_T \) is thermal diffusivity in horizontal direction, \( \kappa_S \) is thermal diffusivity in vertical direction, \( \delta, \omega \) are the amplitude and frequency of modulation. Equation (2.5) represents the externally imposed gravity field.
The thermo-rheological relationship in Eq. (2.7) is guided by Nield [1]. The thermal boundary conditions are

\[ T = T_0 + \Delta T \quad \text{at} \quad z = 0 \quad \text{and} \quad T = T_0 \quad \text{at} \quad z = d. \]  

(2.8)

Since an uniform concentration gradient \( \frac{\Delta S}{d} \), has been maintained between the walls of the liquid layer, therefore the boundary conditions on \( S \) are

\[ S = S_0 + \Delta S \quad \text{at} \quad z = 0 \quad \text{and} \quad S = S_0 \quad \text{at} \quad z = d. \]  

(2.9)

The basic state temperatures and concentration satisfy the equations

\[ \frac{d^2 S_b}{dz^2} = 0, \quad \frac{d^2 T_b}{dz^2} = 0. \]  

(2.11)

The conduction state solutions of the Eq. (2.11), subject to the boundary conditions Eqs. (2.8-2.9) are given by

\[ S_b(z) = S_0 + \Delta S \left( 1 - \frac{z}{d} \right), \]  

(2.12)

\[ T_b(z) = T_0 + \Delta T \left( 1 - \frac{z}{d} \right). \]  

(2.13)

We assume finite amplitude perturbations to the basic state in the form

\[ \tilde{\psi} = \tilde{\psi}_b + \tilde{\psi}', \quad T = T_b + T', \quad S = S_b + S', \quad p = p_b + p', \quad \rho = \rho_b + \rho'. \]  

(2.14)

Substituting Eq. (2.14) in the set of Eqs. (2.1)-(2.7), we get the following equations

\[ \nabla \tilde{\psi} = 0, \]  

(2.15)

\[ \rho_0 \left( \frac{\partial \tilde{\psi}}{\partial t} + (\tilde{\psi} \nabla) \tilde{\psi} \right) = -\nabla p' + \rho' \tilde{g}(t) \tilde{k} + \mu(T) \nabla^2 \tilde{\psi}, \]  

(2.16)

\[ \gamma \frac{\partial T'}{\partial t} + (\tilde{\psi} \nabla) T' + w' \frac{\partial T_b}{\partial z} = \kappa_T \nabla^2 T', \]  

(2.17)

\[ \phi \frac{\partial S'}{\partial t} + (\tilde{\psi} \nabla) S' + w' \frac{d S_b}{d z} = \kappa_S \nabla^2 S', \]  

(2.18)

\[ \rho' = -\rho_0 \left[ \beta_T T' - \beta_S S' \right]. \]  

(2.19)

We consider only two-dimensional disturbances in our study, and hence the stream function \( \psi \) is introduced as

\[ u' = \frac{\partial \psi}{\partial z}, \quad w' = -\frac{\partial \psi}{\partial x}. \]  

(2.20)

By operating curl twice on Eq. (2.16), we eliminate \( p' \) from it, and use Eq. (2.19) to eliminate \( \rho' \), and then render the resulting equation and Eqs. (2.17), (2.18) and (2.20) dimensionless using the following transformations

\[ \psi = \kappa_T \psi^*, (x, y, z) = d (x^*, y^*, z^*), \quad T = \Delta T \quad T^*, \quad S = \Delta S \quad S^*, \quad t = \frac{d^2 T^*}{T}. \]

We obtain the non-dimensional governing equations in the form (on dropping the asterisks for simplicity)

\[ \frac{1}{Pr} \frac{\partial}{\partial t} (\nabla^2 \psi) - \frac{1}{Pr} \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} = \frac{\partial T}{\partial (x, z)} + \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} - Ra_T (1 + g_m) \frac{\partial T}{\partial x} \]  

\[ + Ra_S (1 + g_m) \frac{\partial S}{\partial x}. \]  

(2.21)

\[ \frac{\partial \psi}{\partial x} - \nabla^2 T = \frac{\partial T}{\partial t} + \frac{\partial (\psi, T)}{\partial (x, z)}, \]  

(2.22)

\[ \frac{\partial S}{\partial x} - \frac{1}{Le} \nabla^2 S = \frac{\partial S}{\partial t} + \frac{\partial (\psi, S)}{\partial (x, z)}, \]  

(2.23)
where $g_m = \epsilon^2 \cos(\Omega t)$ and $\overline{p}(T) = \frac{1}{1 - \epsilon^2 \sqrt{V_T}}$, $\epsilon^2$ is a small quantity which indicates that the viscosity variation with temperature is weak, $V$ is the temperature dependent viscosity. The non-dimensionalized parameters in the above equations are $Pr = \frac{\nu_0}{\kappa T}$ is the Prandtl number, $Ra_T = \frac{\beta_0 \Delta T d_T^4}{\nu_0 k T}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_0 \Delta S d_T^4}{\kappa S}$ is the solute Rayleigh number, and $Le = \frac{\kappa}{k S}$ is the Lewis number.

To keep the time variation slow, we have re-scaled the time $t$ by using the time scale $\tau = \epsilon^2 t$. Now, to study the stationary mode of double-diffusive convection, we write the above non-linear system Eqs. (2.21)-(2.23) into the matrix form

$$
\begin{bmatrix}
-\overline{p}(T) \nabla^4 & Ra_T \frac{\partial}{\partial x} & -Ra_S \frac{\partial}{\partial x}

0 & -\nabla^2 & 0

\frac{\partial}{\partial x} & 0 & -\frac{1}{Le} \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi

T

S
\end{bmatrix} =
\begin{bmatrix}
-g_m (Ra_T \frac{\partial}{\partial x} - Ra_S \frac{\partial}{\partial x}) + \frac{\partial p(T)}{\partial x} \frac{\partial \psi}{\partial x}

-\epsilon^2 \frac{\partial^2}{\partial x^2} (\nabla^2 \psi) + \frac{\partial p(T)}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x,z)}

-\epsilon^2 \frac{\partial T}{\partial x} + \frac{\partial (\psi, T)}{\partial (x,z)}

-\epsilon^2 \frac{\partial S}{\partial x} + \frac{\partial (\psi, S)}{\partial (x,z)}
\end{bmatrix}.

(2.24)

We solve Eq. (2.24) by using $\mu = \mu(T_b)$ guided by Nield and considering free-free, isothermal and isohaline boundary conditions as given below

$$
\psi = 0 = \nabla^2 \psi, \quad T = S = 1 \text{ on } z=0,
$$

$$
\psi = 0 = \nabla^2 \psi, \quad T = S = 0 \text{ on } z=1,
$$

(2.25)

### 3 Heat and mass transport for stationary instability

To solve the system (2.24), we introduce the following asymptotic expansion

$$
Ra_T = Ra_{oc} + \epsilon^2 R_2 + \epsilon^4 R_4 + \ldots,
$$

(3.26)

$$
\psi = \epsilon \psi_1 + \epsilon^2 \psi_2 + \epsilon^3 \psi_3 + \ldots,
$$

(3.27)

$$
T = \epsilon T_1 + \epsilon^2 T_2 + \epsilon^3 T_3 + \ldots,
$$

(3.28)

$$
S = \epsilon S_1 + \epsilon^2 S_2 + \epsilon^3 S_3 + \ldots,
$$

(3.29)

where $Ra_{oc}$ is the critical value of the Rayleigh number at which the onset of convection takes place in the absence of temperature modulation.

**At the lowest order**, we have

$$
\begin{bmatrix}
-\nabla^4 & Ra_{oc} \frac{\partial}{\partial x} & -Ra_S \frac{\partial}{\partial x}

0 & -\nabla^2 & 0

\frac{\partial}{\partial x} & 0 & -\frac{1}{Le} \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_1

T_1

S_1
\end{bmatrix} =
\begin{bmatrix}
0

0

0
\end{bmatrix}.

(3.30)

The solution of the lowest order system is

$$
\psi_1 = \frac{\alpha^2}{k_c^2 + \alpha^2} \sin \left( \frac{k_c x}{\alpha^2} \right) \sin \left( \frac{\pi z}{\alpha} \right),
$$

$$
T_1 = -\frac{k_c}{\alpha^2} \cos \left( \frac{k_c x}{\alpha^2} \right) \sin \left( \frac{\pi z}{\alpha} \right),
$$

$$
S_1 = -\frac{k_c Le}{\alpha^2} \cos \left( \frac{k_c x}{\alpha^2} \right) \sin \left( \frac{\pi z}{\alpha} \right).
$$

(3.31)

where $\alpha^2 = k_c^2 + \pi^2$. The system (2.24) gives us the critical value of the Rayleigh number and the corresponding wave number for the onset of stationary convection

$$
Ra_{oc} = \frac{\alpha^6 + Ra_S Le k_c^2}{k_c^2},
$$

(3.32)
\( k_c = \frac{\pi}{\sqrt{2}} \)  

(3.32)

which are the results given by Chandrasekhar [25].

At the second order we have

\[
\begin{bmatrix}
-\nabla^4 & R_{0c} \frac{\partial}{\partial x} & -RaS \frac{\partial}{\partial x} \\
\frac{\partial}{\partial x} & -\nabla^2 & 0 \\
0 & 0 & -\frac{1}{Le} \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_2 \\
T_2 \\
S_2
\end{bmatrix} =
\begin{bmatrix}
R_{21} \\
R_{22} \\
R_{23}
\end{bmatrix}
\]

(3.33)

where

\begin{align*}
R_{21} &= 0, \\
R_{22} &= \frac{\partial \psi_1}{\partial x} \frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial T_1}{\partial x}, \\
R_{23} &= \frac{\partial \psi_1}{\partial x} \frac{\partial S_1}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial S_1}{\partial x}.
\end{align*}

(3.34) (3.35) (3.36)

The second order solution can be obtained as follows

\begin{align*}
\psi_2 &= 0, \\
T_2 &= \frac{k_c^2 A^2}{8\pi\alpha^2} \sin(2\pi z), \\
S_2 &= -\frac{k_c^2 Le^2 A^2}{8\pi\alpha^2} \sin(2\pi z).
\end{align*}

(3.37) (3.38) (3.39)

The horizontally-averaged Nusselt number, \( Nu \), and Sherwood number, \( Sh \), for the stationary mode of double-diffusive convection (the preferred mode in this problem) are given by

\[
Nu(\tau) = 1 + \frac{k_c}{2\pi} \int_0^{2\pi} \left( \frac{\partial T_b}{\partial z} \right) dx, \\
Sh(\tau) = 1 + \frac{k_c}{2\pi} \int_0^{2\pi} \left( \frac{\partial S_b}{\partial z} \right) dx,
\]

\[
Nu(\tau) = 1 + \frac{k_c^2}{4\alpha^2} [A(\tau)]^2, \\
Sh(\tau) = 1 + \frac{k_c^2 Le^2}{4\alpha^2} [A(\tau)]^2.
\]

(3.40) (3.41)

At the third order, we have

\[
\begin{bmatrix}
-\nabla^4 & R_{0c} \frac{\partial}{\partial x} & -RaS \frac{\partial}{\partial x} \\
\frac{\partial}{\partial x} & -\nabla^2 & 0 \\
0 & 0 & -\frac{1}{Le} \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_3 \\
T_3 \\
S_3
\end{bmatrix} =
\begin{bmatrix}
R_{31} \\
R_{32} \\
R_{33}
\end{bmatrix}
\]

(3.42)

\begin{align*}
R_{31} &= -\frac{1}{Pr} \frac{\partial}{\partial \tau} \left( \nabla^2 \psi_1 \right) - VT_b \left( \nabla^2 \psi_1 \right) + V \frac{\partial}{\partial z} \left( \nabla^2 \psi_1 \right) - \left( R_{0c}(2VT_b + gm) + R_2 \right) \frac{\partial T_1}{\partial x} \\
&+ RaS(2VT_b + gm) \frac{\partial S_1}{\partial x}, \\
R_{32} &= \frac{\partial \psi_1}{\partial x} \frac{\partial T_2}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial T_2}{\partial x}, \\
R_{33} &= \frac{\partial \psi_1}{\partial x} \frac{\partial S_2}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial S_2}{\partial x}.
\end{align*}
Using first and second order solutions we can easily obtain the expression for $R_{31}$-$R_{33}$. Now applying solvability condition for the existence of third order solution, we obtain the Ginzburg-Landau equation for stationary convection with time periodic coefficients in the form

$$A_1 \frac{dA(\tau)}{d\tau} = A_2 A(\tau) - A_3 A^3(\tau),$$

(3.43)

where

\[ A_1 = \left[ \frac{k^2}{\alpha^2} + \frac{Ra k^2}{\alpha^2} - \frac{Ra_S k^2 L e^2}{\alpha^3} \right], \]

\[ A_2 = \frac{k^2}{\alpha^2} \left[ R_2 + VR_{0c} - \frac{V \alpha^4}{\kappa^2} - Ra_S V L e + \delta \cos(\Omega \tau) \left( R_{0c} - Ra_S L e \right) \right], \]

\[ A_3 = \frac{k^4}{8 \kappa^4} \left[ R_{0c} - Ra_S L e^3 \right]. \]

The solution of Eq. (3.43), subject to the initial condition $A(0) = a_0$, where $a_0$ is a chosen initial amplitude of convection, can be obtained by using NDSolve, Mathematica 8. In calculations we may assume $R_2 = R_{0c}$, to keep the parameters to the minimum.

4 Analytical solution for Unmodulated case

In the case of unmodulated fluid layer, the above Ginzburg-Landau equation can be written as

$$A_1 A_u'(\tau) - A_2 A_u(\tau) + A_3 A_u^3(\tau) = 0,$$

(4.44)

where $A_u(\tau)$ is an amplitude of convection for unmodulated case and $A_1, A_3$ have the same expression as given in the Eq. (3.43), and $A_2 = \frac{k^2}{\alpha^2} \left[ R_2 + VR_{0c} - \frac{V \alpha^4}{\kappa^2} - Ra_S V L e \right].$

The solution of Eq. (4.44), is given by

$$A_u(\tau) = \frac{1}{\sqrt{\left( \frac{A_1}{A_2} + C_1 \exp \left[ \frac{-2A_2}{A_1} \right] \right)}},$$

(4.45)

where $C_1$ is a parameter, it can be obtained for given suitable initial condition. The horizontal averaged Nusselt and Sherwood numbers in this case is obtained, respectively from Eq. (3.40) and Eq. (3.41) by using the value of $A_u(\tau)$ in the place of $A(\tau)$. It can also be done in the case of gravity modulation by the similar process.

5 Results and Discussion

The present paper deals with double-diffusive convection under gravity modulation by using Ginzburg-Landau model. It is necessary to consider a non-linear theory to analyze heat and mass transfer which is not possible by the linear theory. Also, we consider the effect of gravity modulation to be of order $O(\epsilon^2)$, this leads to small amplitude of modulation. Such an assumption will help us in obtaining the amplitude equation of convection in a rather simple and elegant manner and is much easier to obtain than in the case of the Lorenz model. Also, we have investigated the effect of thermo-rheological parameter on heat and mass transfer.

The parameters that arise in the problem are $Pr, V, Le, Ra_S, \delta, \Omega$, these parameters influence the convective heat and mass transfer. The first four parameters related to the fluid layer, and the last two concern the external mechanisms of controlling convection. Also the fluid layer is not considered to be highly viscous, therefore only moderate values of $Pr$ are taken for calculations. The effect of gravity modulation is represented by an amplitude $\delta$, which takes the values around 0.3, since we are studying the effect of small amplitude modulation on the heat and mass transport.

Further, as the effect of low frequencies on the onset of convection as well as on the heat and mass transport is maximum, therefore the modulation of gravity is assumed to be of low frequency. It is important at this stage to consider the effect of $Pr, V, Le, Ra_S, \delta, \Omega$ on the onset of convection. The temperature dependent viscous parameter $V$ is guided by [9]. Heat and mass transfer quantified by the Nusselt and Sherwood numbers from (3.40)-(3.41). Figs. [2,3] show the individual effect of each non-dimensional parameter on heat and mass transfer.
Fig. 2: Nu versus $\tau$ (a) Pr  (b) V  (c) Le  (d) Ra$_S$  (e) $\delta$  (f) $\Omega$. 
i. The effect of Prandtl number $Pr$ for small values of time $\tau$ is to advance the convection and hence heat and mass transfer, given in Figs. [2a], [3a], however for large values of time it shows oscillatory behaviour.

ii. The effects of thermo-rheological parameter $V$ and Lewis $Le$ number is to advance the convection and hence heat and mass transfer. Thus, both $V$ and $Le$ have destabilizing effect on the system, given in Figs [2b, 2c], [3b, 3c].

iii. The effect of solutal Rayleigh number $Ra_s$ is to increase $Nu$ and $Sh$, so the heat and mass transfer. Hence, it has destabilizing effect given by the Figs [2d], [3d]. Though the presence of a stabilizing
gradient of solute will prevent the onset of convection, the strong finite-amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. As a consequence, the inhibiting effect of the solute gradient is greatly reduced and hence fluid will convect more and more heat and mass, when $Ra_S$ is increased.

iv. An increment in amplitude of g-jitter $\delta$ leads to increment in $Nu$ and $Sh$, hence heat and mass transfer as given in Figs. [2e]$_{Nu}$, [3e]$_{Sh}$.

v. An increment in frequency of modulation shortens the wavelength and decreases the magnitude of $Nu$, $Sh$, and hence heat and mass transfer as given in Figs [2f]$_{Nu}$, [3f]$_{Sh}$.

vi. The comparison between modulated and unmodulated system is shown in Fig.[4a,b], unmodulated case has no effect on heat and mass transfer except for small values of time. But, modulated case has an oscillatory behaviour of heat and mass transfer.

vii. Variation of stream lines, isotherms, isohalines at different instant of time is shown graphically in Figs. [5, 7]. From Figs. [5a-f], it is clear that the magnitudes of stream lines increase as time increases. Figs. [6a-f] show the variation of isotherms at different instant of time, and found that initially isotherms are parallel showing that heat transport is only by conduction. As time increases isotherms start oscillating, showing that convective regime is in place, and then form contours as convection becomes more vigorous. Similar behaviour is also observed for isohalines in Fig.[7a,f]. However, it is clear from the Figs. [5, 7] that, after reaching at some instant of time, there is no change in stream lines, isotherms and isohalines, showing that study state has been achieved.
Fig. 5: Stream lines for $\tau$ (a) 0.0 (b) 0.1 (c) 0.3 (d) 0.6 (e) 1.0 (f) 2.0.
**Fig. 6**: Isotherms for \( \tau \) (a) 0.0 (b) 0.1 (c) 0.3 (d) 0.6 (e) 1.0 (f) 2.0.
Fig. 7: Isohalines for $\tau$ (a) 0.0 (b) 0.1 (c) 0.3 (d) 0.6 (e) 1.0 (f) 2.0.
viii. The results of this work can be summarized as follows from the Figs. [2, 3].
1. \([\text{Nu}/\text{Sh}]_{Pr=0.5} < [\text{Nu}/\text{Sh}]_{Pr=1.0} < [\text{Nu}/\text{Sh}]_{Pr=1.5}\)
2. \([\text{Nu}/\text{Sh}]_{V=0.2} < [\text{Nu}/\text{Sh}]_{V=0.4} < [\text{Nu}/\text{Sh}]_{V=0.7}\)
3. \([\text{Nu}]_{Le=1.2} < [\text{Nu}]_{Le=1.6} < [\text{Nu}]_{Le=2.2}\)
4. \([\text{Sh}]_{Le=1.2} < [\text{Sh}]_{Le=1.3} < [\text{Sh}]_{Le=1.4}\)
5. \([\text{Nu}/\text{Sh}]_{RaS=20} < [\text{Nu}/\text{Sh}]_{RaS=50} < [\text{Nu}/\text{Sh}]_{RaS=100}\)
6. \([\text{Nu}/\text{Sh}]_{\delta=0.3} < [\text{Nu}/\text{Sh}]_{\delta=0.5} < [\text{Nu}/\text{Sh}]_{\delta=0.7}\)
7. \([\text{Nu}/\text{Sh}]_{\Omega=100} < [\text{Nu}/\text{Sh}]_{\Omega=50} < [\text{Nu}/\text{Sh}]_{\Omega=30} < [\text{Nu}/\text{Sh}]_{\Omega=2}\).

6 Conclusions

We performed the weak nonlinear analysis, using the Ginzburg-Landau equation for double diffusive convection, in a temperature sensitive horizontal fluid layer, in the presence of gravity modulation. The following conclusions have been made from our analysis.

i. Prandtl number \(Pr\) has an effect for small values of time, further increasing time no effect on heat and mass transfer.

ii. Thermo-rheological parameter \(V\): both heat and mass transfer increase.

iii. Lewis number \(Le\): heat and mass transfer increase.

iv. Solute Rayleigh number \(Ra_S\): heat and mass transfer increase.

v. amplitude of modulation \(\delta\): heat and mass transfer increase.

vi. The frequency of modulation \(\Omega\): heat and mass transfer decrease.

vii. The unmodulated system has no effect on heat and mass transfer but modulated system affect the system.

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