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Shortest path with complement of type-2 fuzzy number

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Abstract

This paper presents a study of fuzzy shortest path with complement of type-2 fuzzy number on a network. A proposed algorithm also gives the shortest path length using ranking function from source node to destination node. Here each arc length is assigned to complement of type-2 fuzzy number. An illustrative example also included to demonstrate our proposed algorithm.

Keywords: Type-2 fuzzy set, Complement of type-2 fuzzy set, Type-2 fuzzy number, Ranking function, footprint of uncertainty.

2010 MSC: 94D05

1 Introduction

The concept of a type-2 fuzzy set was introduced by Zadeh [12] as an extension of the concept of an ordinary fuzzy set. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets. They are very useful for incorporating linguistic uncertainties, for example the words that are used in linguistic knowledge can mean different things to different people [4].

Type-2 fuzzy sets have already been used in a number of applications, including decision making, solving fuzzy relation equations, and pre-processing of data. Type-2 fuzzy logic has gained much attention recently due to its ability to handle uncertainty, and many advances appeared in both theory and applications. According to John, "Type-2 fuzzy sets allow for linguistic grades of membership, thus assisting in knowledge representation and they also offer improvement on inferencing with type-1 sets [2]". Dinagar and Anbalagan [10] presented new ranking function and arithmetic operations on generalized type-2 trapezoidal fuzzy numbers.

The fuzzy shortest path problem was first analyzed by Dubois and Prade [1]. Okada and Soper [9] developed an algorithm based on the multiple labeling approach, by which a number of nondominated paths can be generated.

In conventional shortest path problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in real life situations, there always exist uncertainty about the parameters between different nodes. In such cases the parameters are requested by fuzzy numbers. However, type-1 fuzzy weights are not good enough. For this situation, we need type-2 fuzzy weights instead of type-1 fuzzy weights to show the variations.

This paper is organized as follows: In section 2,we have some basic concepts required for analysis. Section 3, gives an algorithm for finding shortest path and Shortest path length from source node to destination node using complement of type-2 fuzzy number. Section 4, gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5.

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2 Concepts

In this section some basic concepts of type-2 fuzzy set and ranking function are reviewed.

2.1 Type-2 Fuzzy Set:

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$. ie., $\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u))/\forall\ x \in X,\ u \in Jx \subseteq [0,1]\}$ in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. $\tilde{A}$ can be expressed as $\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u)\ J_x \subseteq [0,1]$, where denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2 Complement of Type-2 Fuzzy Set:

The complement of the type-2 fuzzy set having a fuzzy membership function given in the following formula.

$$\overline{\tilde{A}} = \sum_{x \in X} \left[\sum_{u \in J_x} f_x(u)/(1-u) \right] /x$$

2.3 Interval Type-2 Fuzzy Set:

Interval type-2 fuzzy set is defined to be a T2FS where all its secondary grade are of unity for all $f_x(u) = 1$.

2.4 Footprint Of Uncertainty:

Uncertainty in the primary membership of a type-2 fuzzy set, $\tilde{A}$, consists of a bounded region that we call the footprint of uncertainty (FOU). It is the union of all primary membership.

$$ie., \quad FOU(\tilde{A}) = \int_{x \in X} J_x$$

The FOU can be described in terms of its upper and lower membership function.

$$FOU(\tilde{A}) = \int_{x \in X} [\mu_{\tilde{A}}(x)^l, \mu_{\tilde{A}}(x)^u].$$

2.5 Principal Membership Function:

The principal membership function defined as the union of all the primary membership having secondary grades equal to 1.

$$ie., P_r(\tilde{A}) = \int_{x \in X} u/x | f_x(u) = 1.$$

2.6 Type-2 Fuzzy Number:

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

i. $\tilde{A}$ is normal,

ii. $\tilde{A}$ is a convex set,

iii. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.7 Normal Type-2 Fuzzy Number:

A T2FS, $\tilde{A}$ is said to be normal if its FOU is normal IT2FS and it has a primary membership function.

2.8 Extension Principle

Let $A_1, A_2, ..., A_r$ be type-1 fuzzy sets in $X_1, X_2, ..., .X_r$, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, ..., A_r$ a type-1 fuzzy set B on Y, through f, i.e, $B = f(A_1, ..., A_r)$, such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1, x_2, ..., x_n \in f^{-1}(y)} min\{\mu_{A_1}(x_1), ..., \mu_{A_n}(x_n)\} & \text{if} \quad f^{-1}(y) \neq \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.9 Addition On Type-2 Fuzzy Numbers:

$$\begin{aligned} \text{Let} \quad \tilde{A} &= \bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{A}_{\tilde{\alpha}}) \\ &= (A^L, A^M, A^N, A^U) \\ &= ((a_1^L, a_2^L, a_3^L, a_4^L; \lambda_A), (a_1^M, a_2^M, a_3^M, a_4^M), (a_1^N, a_2^N, a_3^N, a_4^N), (a_1^U, a_2^U, a_3^U, a_4^U)) \end{aligned}$$

and

$$\begin{aligned} \text{Let} \quad \tilde{B} &= \bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{B}_{\tilde{\alpha}}) \\ &= (B^L, B^M, B^N, B^U) \\ &= ((b_1^L, b_2^L, b_3^L, b_4^L; \lambda_B), (b_1^M, b_2^M, b_3^M, b_4^M), (b_1^N, b_2^N, b_3^N, b_4^N), (b_1^U, b_2^U, b_3^U, b_4^U)) \end{aligned}$$

be two normal type-2 fuzzy numbers. By using extension principle we have

$$\begin{aligned} \tilde{A} + \tilde{B} &= \left[\bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{A}_{\tilde{\alpha}}) \right] + \left[\bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{B}_{\tilde{\alpha}}) \right] \\ &= (A^L + B^L, A^M + B^M, A^N + B^N, A^U + B^U) \\ \tilde{A} + \tilde{B} &= ((a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; min\{\lambda_A, \lambda_B\}), \\ &\quad (a_1^M + b_1^M, a_2^M + b_2^M, a_3^M + b_3^M, a_4^M + b_4^M), (a_1^N + b_1^N, a_2^N + b_2^N, a_3^N + b_3^N, a_4^N + b_4^N), \\ &\quad (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U)) \end{aligned}$$

2.10 Ranking Function On Type-2 Fuzzy Number:

Let $\tilde{A} = ((a_1^L, a_2^L, a_3^L, a_4^L; \lambda_A), (a_1^M, a_2^M, a_3^M, a_4^M), (a_1^N, a_2^N, a_3^N, a_4^N), (a_1^U, a_2^U, a_3^U, a_4^U))$ be a type-2 normal trapezoidal fuzzy number, then the ranking function is defined as

$$\begin{aligned} R(\tilde{A}) = (&a_1^L + 2a_2^L + 2a_3^L + a_4^L + 2a_1^M + 4a_2^M + 4a_3^M + 2a_4^M + 2a_1^N + 4a_2^N + 4a_3^N + \\ &2a_4^N + a_1^U + 2a_2^U + 2a_3^U + a_4^U)/36. \end{aligned}$$

3 Algorithm

Step 1 : Form the possible paths from starting node to destination node.

Step 2 : Find the paths length $\tilde{L}_i$ with complement of type-2 fuzzy number, $i = 1, 2, ...n$ for possible n paths.

Step 3 : Compute

$$\begin{aligned} R(\overline{\tilde{L}}_i) = (&a_1^L + 2a_2^L + 2a_3^L + a_4^L + 2a_1^M + 4a_2^M + 4a_3^M + 2a_4^M + 2a_1^N + 4a_2^N + 4a_3^N + \\ &2a_4^N + a_1^U + 2a_2^U + 2a_3^U + a_4^U)/36. \end{aligned}$$

Step 4 : Decide the shortest path for complement of type-2 fuzzy number with minimum rank.

4 Network Terminology

Consider a directed network $G(V,E)$ consisting of a finite set of nodes $V = \{1,2,...n\}$ and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where $i,j \in V$ and $i \neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence $P_{ij} = \{i = i_1, (i_1,i_2), i_2, ..., i_{l-1}, (i_{l-1}, i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{si} in $G(V,E)$ is assume for every node $i \in V - \{s\}$.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{i,j \in P} \tilde{d}_{ij}$.

5 Numerical Example

The problem is to find the shortest path and shortest path length between source node and destination node in the network with complement of type-2 fuzzy number.

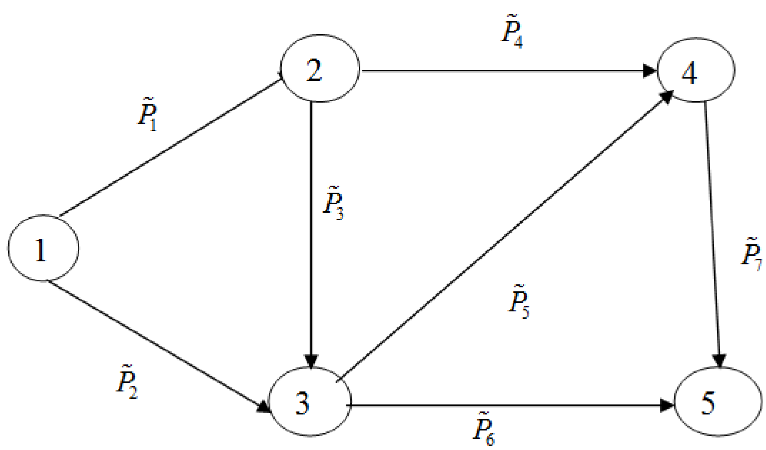

Fig 5.1

In this network each edge have been assigned to type-2 fuzzy number as follows:

$\tilde{P}_1 = ((0.5, 0.7, 0.8, 0.9; 0.4), (0.4, 0.6, 0.7, 0.9), (0.2, 0.3, 0.5, 0.8), (0.1, 0.5, 0.7, 1.0))$

$\tilde{P}_2 = ((0.4, 0.5, 0.7, 1.0; 0.5), (0.3, 0.5, 0.7, 0.9), (0.3, 0.5, 0.7, 0.8), (0.2, 0.4, 0.6, 0.9))$

$\tilde{P}_3 = ((0.6, 0.8, 0.9, 0.9; 0.7), (0.5, 0.6, 0.7, 0.9), (0.3, 0.5, 0.6, 0.8), (0.1, 0.2, 0.3, 0.4))$

$\tilde{P}_4 = ((0.4, 0.5, 0.5, 0.9; 0.6), (0.3, 0.5, 0.6, 0.9), (0.3, 0.4, 0.6, 0.8), (0.2, 0.4, 0.5, 0.7))$

$\tilde{P}_5 = ((0.6, 0.7, 0.7, 0.9; 0.3), (0.4, 0.6, 0.7, 0.9), (0.4, 0.6, 0.9, 1.0), (0.3, 0.5, 0.8, 0.9))$

$\tilde{P}_6 = ((0.7, 0.8, 0.8, 0.9; 0.4), (0.5, 0.7, 0.9, 1.0), (0.3, 0.5, 0.7, 0.9), (0.2, 0.3, 0.5, 0.8))$

$\tilde{P}_7 = ((0.5, 0.6, 0.8, 1.0; 0.2), (0.3, 0.5, 0.8, 1.0), (0.2, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6))$

Complements of $\tilde{P}_1, \tilde{P}_2, ...\tilde{P}_7$ are

$\overline{\tilde{P}_1} = ((0.5, 0.3, 0.2, 0.1; 0.4), (0.6, 0.4, 0.3, 0.1), (0.8, 0.7, 0.5, 0.2), (0.9, 0.5, 0.3, 0.0))$

$\overline{\tilde{P}_2} = ((0.6, 0.5, 0.3, 0.0; 0.5), (0.7, 0.5, 0.3, 0.1), (0.7, 0.5, 0.3, 0.2), (0.8, 0.6, 0.4, 0.1))$

$\overline{\tilde{P}_3} = ((0.4, 0.2, 0.1, 0.1; 0.7), (0.5, 0.4, 0.3, 0.1), (0.7, 0.5, 0.4, 0.2), (0.9, 0.8, 0.7, 0.6))$

$\overline{\tilde{P}_4} = ((0.6, 0.5, 0.5, 0.1; 0.6), (0.7, 0.5, 0.4, 0.1), (0.7, 0.6, 0.4, 0.2), (0.8, 0.6, 0.5, 0.3))$

$\overline{\tilde{P_5}} = ((0.4, 0.3, 0.3, 0.1; 0.3), (0.6, 0.4, 0.3, 0.1), (0.6, 0.4, 0.1, 0.0), (0.7, 0.5, 0.2, 0.1))$

$\overline{\tilde{P_6}} = ((0.3, 0.2, 0.2, 0.1; 0.4), (0.5, 0.3, 0.1, 0.0), (0.7, 0.5, 0.3, 0.1), (0.8, 0.7, 0.5, 0.2))$

$\overline{\tilde{P_7}} = ((0.5, 0.4, 0.2, 0.0; 0.2), (0.7, 0.5, 0.2, 0.0), (0.8, 0.4, 0.2, 0.1), (0.8, 0.6, 0.5, 0.4))$

Solution :

Step 1: Form the possible paths from starting node to destination node

Possible paths are $1-2-4-5, 1-2-3-4-5, 1-2-3-5, 1-3-5, 1-3-4-5$

Step 2: Find the path legth $\tilde{L_i}$ with complement of type-2 fuzzy number, $i = 1, 2, ...n$ for possible n paths.

$$\begin{aligned}
\text{Path lengths are}\quad & \overline{\tilde{L_1}} = 1-2-4-5 = \tilde{P_1} + \tilde{P_4} + \tilde{P_7} \\
& \overline{\tilde{L_2}} = 1-2-3-4-5 = \tilde{P_1} + \tilde{P_3} + \tilde{P_5} + \tilde{P_7} \\
& \overline{\tilde{L_3}} = 1-2-3-5 = \tilde{P_1} + \tilde{P_3} + \tilde{P_6} \\
& \overline{\tilde{L_4}} = 1-3-5 = \tilde{P_2} + \tilde{P_6} \\
& \overline{\tilde{L_5}} = 1-3-4-5 = \tilde{P_2} + \tilde{P_5} + \tilde{P_7}
\end{aligned}$$

$$\begin{aligned}
\overline{\tilde{L_1}} =& 0.5+0.6+0.5, 0.3+0.5+0.4, 0.2+0.5+0.2, 0.1+0.1+0.0; min\,\{0.4, 0.6, 0.2\}), \\
& (0.6+0.7+0.7, 0.4+0.5+0.5, 0.3+0.4+0.2, 0.1+0.1+0.0), \\
& (0.8+0.7+0.8, 0.7+0.6+0.4, 0.5+0.4+0.2, 0.2+0.2+0.1), \\
& (0.9+0.8+0.8, 0.5+0.6+0.6, 0.3+0.5+0.5, 0.0+0.3+0.4)
\end{aligned}$$

$$\overline{\tilde{L_1}} = ((1.6, 1.2, 0.9, 0.2; 0.2), (2.0, 1.4, 0.9, 0.2), (2.3, 1.7, 1.1, 0.5), (2.5, 1.7, 1.3, 0.7))$$

In the same way, we get the path leghts are as follows:

$$\begin{aligned}
\overline{\tilde{L_2}} &= ((1.8, 1.2, 0.8, 0.3; 0.2), (2.4, 1.7, 0.9, 0.3), (2.9, 2.0, 1.2, 0.5), (3.2, 2.4, 1.7, 1.1)) \\
\overline{\tilde{L_3}} &= ((1.2, 0.7, 0.5, 0.3; 0.4), (1.6, 1.1, 0.9, 0.2), (2.2, 1.7, 1.2, 0.5), (2.6, 2.0, 1.5, 0.8)) \\
\overline{\tilde{L_4}} &= ((0.9, 0.7, 0.5, 0.1; 0.4), (1.2, 0.8, 0.4, 0.1), (1.4, 1.0, 0.6, 0.3), (1.6, 1.3, 0.9, 0.5)) \\
\overline{\tilde{L_5}} &= ((1.5, 1.2, 0.8, 0.1; 0.2), (2.0, 1.4, 0.8, 0.2), (2.1, 1.3, 0.6, 0.3), (2.3, 1.7, 1.1, 0.6))
\end{aligned}$$

Step 3 : Compute

$$\begin{aligned}
R(\overline{\tilde{L_i}}) = (& a_1^L + 2a_2^L + 2a_3^L + a_4^L + 2a_1^M + 4a_2^M + 4a_3^M + 2a_4^M + 2a_1^N + 4a_2^N + 4a_3^N + 2a_4^N + a_1^U \\
& + 2a_2^U + 2a_3^U + a_4^U)/36.
\end{aligned}$$

$$\begin{aligned}
R(\overline{\tilde{L_1}}) &= 1.23 \\
R(\overline{\tilde{L_2}}) &= 1.45 \\
R(\overline{\tilde{L_3}}) &= 1.19 \\
R(\overline{\tilde{L_4}}) &= 0.75 \\
R(\overline{\tilde{L_5}}) &= 1.10
\end{aligned}$$

Step 4: Decide the shortest path for Complement of type-2 fuzzy number with minimum rank.

Path length $\overline{\tilde{L_4}}$ is having the minimum rank $(R(\overline{\tilde{L_4}}) = 0.75)$
The Shortest path length is
$((0.9, 0.7, 0.5, 0.1; 0.4), (1.2, 0.8, 0.4, 0.1), (1.4, 1.0, 0.6, 0.3), (1.6, 1.3, 0.9, 0.5))$

And the corresponding shortest path is

1 - 3 - 5

6 Conclusion

In this paper we proposed an algorithm for finding fuzzy shortest path and shortest path length from source node to destination node on a network using Complement of Type-2 fuzzy number. This proves that it is more flexible and intelligent way in solving problems which deals with *uncertainty and ambiguity* for choosing the best decision since it uses type-2 fuzzy number rather than type-1 fuzzy number.

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