(1, 2)*-rgα-Closed sets in bitopological spaces

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Abstract

In this paper, we introduce (1, 2)*-rgα-closed sets some of its basic properties are studied.

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1 Introduction

N. Levine [1] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Balachandran, Sundaram and Maki [3], Bhattacharyya and Lahiri [4], Arockiarani [2], Dunham [5], Gnanambal [6], Malghan [7], Palaniappan and Rao [8], Park [9] Arya and Gupta [12] and Devi [13], Benchalli and wali [11] have worked on generalized closed sets, their generalizations and related concepts in general topology. In this paper, we define and study the properties of (1,2)*-regular generalized α-closed sets (briefly (1,2)*-rgα-closed) in bitopological spaces.

On comparing topologies on a non-empty set, Weston [14] defined coupling and consistency of the topologies. The beauty is, this study is the possibility of getting always two distinct topologies. Kelly [15] initiated the systematic study of such spaces. That is, a triple(\(X, \tau_1, \tau_2\)), where \(X\) is a nonempty set and \(\tau_1\) and \(\tau_2\) are topologies on \(X\), is called a bitopological space. An year later, various author make their attention in this theory. Fukutake [16] introduced and studied the notions of generalized closed sets in bitopological spaces.

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2 Preliminaries

Throughout the present paper \((X, \tau_1, \tau_2)\), denote bitopological spaces.

**Definition 2.1.** A subset \(A\) of a space \((X, \tau)\) is called

1. a pre-open set [18] if \(A \subset \text{int}(\text{cl}(A))\) and a pre-closed set if \(\text{cl}(\text{int}(A)) \subset A\);

2. a semi-open set[19] if \(A \subset \text{cl}(\text{int}(A))\) and a semi-closed set if \(\text{int}(\text{cl}(A)) \subset A\);

3. a \(\alpha\)-open set[20] if \(A \subset \text{int}(\text{cl}(\text{int}(A)))\) and a \(\alpha\)-closed set if \(\text{cl}(\text{int}(\text{cl}(A))) \subset A\);

4. a semi-preopen set[21] if \(A \subset \text{cl}(\text{int}(\text{cl}(A)))\) and a semi-pre-closed set if \(\text{int}(\text{cl}(\text{int}(A))) \subset A\);

5. a regular open set if \(A=\text{int}(\text{cl}(A))\) and a regular closed set if \(A=\text{cl}(\text{int}(A))\);

6. \(b\)-open [22] or \(sp\)-open [24], \(\gamma\)-open [25] if \(A \subset \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))\).

7. \(\pi\alpha\)-closed [23] if \(\alpha\text{Cl}(A) \subset U\) whenever \(A \subset U\) and \(U\) is \(\pi\)-open in \(X\).

8. \(\pi\gamma\)-closed [26] if \(\gamma\text{Cl}(A) \subset U\) whenever \(A \subset U\) and \(U\) is \(\pi\)-open in \(X\).

**Definition 2.2.** [27] A subset \(A\) of \(X\) is called \(\tau_1\tau_2\)-open if \(S \in \tau_1 \cup \tau_2\) and the complement of \(\tau_1\tau_2\)-open set is \(\tau_1\tau_2\)-closed.

**Definition 2.3.** [27] Let \(A\) be a subset of \(X\)

(i) The \(\tau_1\tau_2\) -closure of \(A\), denoted by \(\tau_1\tau_2\)-cl\((S)\) is defined by \(\cap\{F/S \subset F\text{ and } F\text{ is }\tau_1\tau_2\text{-closed}\}\)

(ii) The \(\tau_1\tau_2\) -interior of \(A\), denoted by \(\tau_1\tau_2\)-int\((S)\) is defined by \(\cup\{F/F \subset S\text{ and } F\text{ is }\tau_1\tau_2\text{-open}\}\)

**Remark 2.1.** [28]

(i) \(\tau_1\tau_2\)-int\((S)\) is \(\tau_1\tau_2\)-open for each \(S \subset X\) and \(\tau_1\tau_2\)-cl\((S)\) is \(\tau_1\tau_2\)-closed for each \(S \subset X\).

(ii) A subset \(S \subset X\) is \(\tau_1\tau_2\)-open iff \(S=\tau_1\tau_2\)-int\((S)\) and \(\tau_1\tau_2\)-closed iff \(S=\tau_1\tau_2\)-cl\((S)\)

(iii) \(\tau_1\tau_2\)-int\((S)\)=int\(\tau_1(S)\) \(\cup\) int\(\tau_2(S)\) and \(\tau_1\tau_2\)-cl\((S)\)=cl\(\tau_1(S)\) \(\cup\) cl\(\tau_2(S)\) for any \(S\subset X\)

(iv) for any family \(\{Si/i \in I\}\) of subsets of \(X\) we have

(a) \(i \cup \tau_1\tau_2 - \text{int}(Si) \subset \tau_1\tau_2 - \text{int}(i \cup_i Si)\)

(b) \(i \cup_i \tau_1\tau_2 - \text{cl}(Si) \subset \tau_1\tau_2 - \text{cl}(i \cup_i Si)\)

(c) \(\tau_1\tau_2 - \text{int}(i \cup_i Si) \subset i \cup_i \tau_1\tau_2 - \text{int}(Si)\)

(d) \(\tau_1\tau_2 - \text{cl}(i \cup_i Si) \subset i \cup_i \tau_1\tau_2 - \text{cl}(Si)\)
(v) \( \tau_1 \tau_2 \)-open sets need not form a topology.

**Definition 2.4.** The finite union of \((1, 2)^*\)-regular open sets [29] is said to be \( \tau_1 \tau_2 - \pi\)-open. The complement of \( \tau_1 \tau_2 - \pi\)-open is said to be \( \tau_1 \tau_2 - \pi\)-closed.

**Definition 2.5.** A subset \( A \) of a bitopological space \((X, \tau_1, \tau_2)\) is called

(i) \((1, 2)^*\)-semi-open set[17] if \( A \subset (X, \tau_1, \tau_2) - \text{cl}(\tau_1 \tau_2 - \text{int}(A)) \)

(ii) \((1, 2)^*\)-preopen set [17] if \( A \subset (X, \tau_1, \tau_2) - \text{int}(\tau_1 \tau_2 - \text{cl}(A)) \)

(iii) \((1, 2)^*\)-\( \alpha\)-open set[17] if \( A \subset \tau_1 \tau_2 - \text{int}(\tau_1 \tau_2 - \text{cl}(A)) \)

(iv) \((1, 2)^*\)-\( \pi\alpha\)-closed [29] if \((1, 2)^*\)-\( \alpha\text{Cl}(A) \subset U \) whenever \( A \subset U \) and \( U \) is \((1, 2)^*\)-\( \pi\)-open in \( X \).

(v) \((1, 2)^*\)-regular open [17] if \( A = \tau_1 \tau_2 - \text{int}(\tau_1 \tau_2 - \text{cl}(A)) \). Complement of the \((1, 2)^*\)-regular open set is called \((1, 2)^*\)-regular closed set.

(vi) \((1, 2)^*\)-b-open [30] if \( A \subset \tau_1 \tau_2 - \text{cl}(\tau_1 - \text{int}(A)) \cup \tau_2 - \text{int}(\tau_1 \tau_2 - \text{cl}(A)) \).

(vii) \((1, 2)^*\)-generalized closed (briefly \((1, 2)^*\)-g-closed)[22] if \( \tau_1 \tau_2 - \text{cl}(A) \subset U \) whenever \( A \subset U \) and \( U \) is \( \tau_1 \tau_2 \)-open in \( X \).

(viii) \((1, 2)^*\)-\( \alpha\)-closure [17] (resp \((1, 2)^*\)-semi-closure) of a subset \( A \) of \( X \), denoted by \((1, 2)^*\)-\( \alpha\text{Cl}(A) \) (resp. \((1, 2)^*\)-\( \alpha\text{sCl}(A) \)) is defined to be the intersection of all \((1, 2)^*\)-\( \alpha\)-closed (resp. \((1, 2)^*\)-\( \alpha\)-semi-closed) sets containing \( A \).

(ix) \((1, 2)^*\)-\( \alpha\)-interior [17] (resp \((1, 2)^*\)-semi-interior) of a subset \( A \) of \( X \) denoted by \((1, 2)^*\)-\( \alpha\text{Int}(A) \) (resp. \((1, 2)^*\)-\( \alpha\text{sInt}(A) \)) is defined to be the union of all \((1, 2)^*\)-\( \alpha\)-open (resp. \((1, 2)^*\)-\( \alpha\)-semi-open) sets containing \( A \).

(x) \((1, 2)^*\)-semi-generalized closed (briefly \((1, 2)^*\)-sg-closed)[17] if \((1, 2)^*\)-\( \alpha\text{sCl}(A) \subset U \) whenever \( A \subset U \) and \( U \) is \((1,2)^*\) semi-open in \( X \).

(xi) \((1, 2)^*\)-generalized semi closed (briefly \((1, 2)^*\)-gs-closed) [17] if \((1, 2)^*\)-\( \alpha\text{gCl}(A) \subset U \) whenever \( A \subset U \) and \( U \) is \((1, 2)^*\) \( \tau_1 \tau_2 \) open in \( X \).

(xii) \((1, 2)^*\)-\( \alpha\)-generalized closed (briefly \((1, 2)^*\)-\( \alpha\)-g-closed) [17] if \((1, 2)^*\)-\( \alpha\text{Cl}(A) \subset U \) whenever \( A \subset U \) and \( U \) is \( \tau_1 \tau_2 \)-open in \( X \).

(xiii) \((1, 2)^*\)-generalized \( \alpha\)-closed (briefly \((1, 2)^*\)-\( \alpha\)-g-closed) [17] if \((1, 2)^*\)-\( \alpha\text{Cl}(A) \subset U \) whenever \( A \subset U \) and \( U \) is \( \tau_1, \tau_2 \)-\( \alpha\)-open in \( X \).

(xiv) \((1, 2)^*\)-weakly generalized closed set (briefly,\((1,2)^*\)-wg-closed)[31] if \((1,2)^*\)-\( \text{clint}(A) \subset U \)
whenever $A \subseteq U$ and $U$ is $\tau_1, \tau_2$ open in $X$.

(1,2)*-weakly closed set (briefly,(1,2)*- w-closed)[31] if $(1,2)*- \text{cl} (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $(1,2)*$-semiopen in $X$.

$(1,2)*$-regular weakly generalized closed set (briefly,$(1,2)*$- rwg-closed) if $(1,2)*- \text{clint} (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $(1,2)*$-regular open in $X$.

3 $(1,2)^{*} - r g_{\alpha}$ closed sets and their basic properties

We introduce the following definition

**Definition 3.1.** A subset $A$ of a space $(X, \tau_1, \tau_2)$ is called $(1,2)^{*}$-regular $\alpha$-open set (briefly,$(1,2)^{*}$-r$\alpha$-open) if there is a$(1,2)^{*}$- regular open set $U$ such that $U \subset A \subset (\tau_1, \tau_2 \alpha \text{cl}(U))$. The family of all $(1,2)^{*}$- regular $\alpha$-open sets of $(X, \tau_1, \tau_2)$ is denoted by $R\alpha O (X, \tau_1, \tau_2)$.

**Definition 3.2.** A subset $A$ of a space $(X, \tau_1, \tau_2)$ is called a $(1,2)^{*}$- regular generalized $\alpha$-closed set (briefly,$(1,2)^{*}$-rg$\alpha$-closed) if $\alpha \text{cl} (A) \subset U$ whenever $A \subset U$ and $U$ is$(1,2)^{*}$- regular $\alpha$-open in $(X, \tau_1, \tau_2)$. We denote the set of all $(1,2)^{*}$-rg$\alpha$-closed sets in $(X, \tau_1, \tau_2)$ by $R\alpha C (X, \tau_1, \tau_2)$. First we prove that the class of $(1,2)^{*}$-rg$\alpha$-closed sets has properly lies between the class of(1,2)*- g$\alpha$-closed sets and the class of$(1,2)^{*}$- regular generalized closed sets.

**Theorem 3.1.** Every $(1,2)^{*}$-g$\alpha$-closed set in $(X, \tau_1, \tau_2)$ is $(1,2)^{*}$- rg$\alpha$-closed set in $(X, \tau_1, \tau_2)$, but not conversely.

**Proof.** The proof follows from the definitions and the fact that every $(1,2)^{*}$- regular open sets is $(1,2)^{*}$-regular $\alpha$-open. The converse of the above theorem need not be true, as seen from the following example.

**Example 3.1.** Let $X=\{a, b, c, d, e\}$ and $\tau_1=\{\phi, \{a, b\}, a, b, c, dX\}$, $\tau_2=\{\phi, \{c, d\}, \{a, b, c, d\}, X\}$, then the set $A=\{a\}$ is $(1,2)^{*}$- rg$\alpha$-closed set but not $(1,2)^{*}$- g$\alpha$-closed set in $(X, \tau_1, \tau_2)$

**Theorem 3.2.** Every$(1,2)^{*}$- w-closed set in $X$ is $(1,2)^{*}$- rg$\alpha$-closed set in $(X, \tau_1, \tau_2)$, but not conversely.

**Proof.** The proof follows from the definitions and the fact that every $(1,2)^{*}$-regular $\alpha$-open set is $(1,2)^{*}$-semiopen and $\tau_1, \tau_2$ closed sets are $\alpha$-closed. The converse of the above theorem need not be true, as seen from the following example.

**Example 3.2.** Let $X=\{a, b, c, d, e\}$ and $\tau_1=\{\phi, \{a, b\}, a, b, c, dX\}$, $\tau_2=\{\phi, \{c, d\}, \{a, b, c, d\}, X\}$, then the set $A=\{b\}$ is $(1,2)^{*}$- rg$\alpha$-closed set but not $(1,2)^{*}$- w-closed set in $(X, \tau_1, \tau_2)$
Theorem 3.3. Every (1,2)*-rw-closed set in \((X, \tau_1, \tau_2)\) is (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\), but not conversely.

Proof. The proof follows from the definitions and the fact that closed sets are (1,2)*-α-closed. The converse of the above theorem need not be true, as seen from the following example.

Example 3.3. Let \(X=\{a, b, c, d, e\}\) and \(\tau_1=\{\phi, \{a, b\}, a, b, c, d, X\}\), \(\tau_2=\{\phi, \{c, d\}, \{a, b, c, d\}, X\}\), then the set \(A=\{b\}\) is (1,2)*-rgα-closed set but not (1,2)*-rw-closed set in \((X, \tau_1, \tau_2)\).

Theorem 3.4. Every (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\) is (1,2)*-rg-closed set in \((X, \tau_1, \tau_2)\), but not conversely.

Proof. The proof follows from the definitions and the fact that every (1,2)*-regular open sets are (1,2)*-regular α-open. The converse of the above theorem need not be true, as seen from the following example.

Example 3.4. Let \(X=\{a, b, c, d, e\}\) and \(\tau_1=\{\phi, \{a, b\}, a, b, c, d, X\}\), \(\tau_2=\{\phi, \{c, d\}, \{a, b, c, d\}, X\}\), then the set \(A=\{a, b\}\) is (1,2)*-rg-closed set but (1,2)*-not rgα-closed set in \((X, \tau_1, \tau_2)\).

Corollary 3.1. Every closed set is (1,2)*-rgα-closed but not conversely.

Corollary 3.2. Every (1,2)*-regular closed set is (1,2)*-rgα-closed but not conversely.

Corollary 3.3. Every (1,2)*-rgα-closed set is a (1,2)*-gpr-closed set but not conversely.

Corollary 3.4. Every (1,2)*-π-closed set is a (1,2)*-rgα-closed set but not conversely.

Theorem 3.5. Every (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\) is (1,2)*-rwg-closed set in \((X, \tau_1, \tau_2)\), but not conversely.

Proof. The proof follows from the definitions and the fact that every (1,2)*-regular open sets are (1,2)*-regular α-open. The converse of the above theorem need not be true, as seen from the following example.

Theorem 3.6. The union of two (1,2)*-rgα-closed subsets of \((X, \tau_1, \tau_2)\) is also (1,2)*-rgα-closed subset of \((X, \tau_1, \tau_2)\).

Proof. Assume that \(A\) and \(B\) are (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\). Let \(U\) be (1,2)*-regular α-open in \((X, \tau_1, \tau_2)\) such that \(A \cup B \subseteq U\). Then \(A \subseteq U\) and \(B \subseteq U\). Since \(A\) and \(B\) are (1,2)*-rgα-closed, \(1,2)*-αcl(A) \subseteq U\) and \((1,2)*-αcl(B) \subseteq U\). Hence\((1,2)*-αcl(A \cup B) = (1,2)*-(αcl(A)) \cup (1,2)*-(αcl(B)) \subseteq U\). That is \(1,2)*-αcl(A \cup B) \subseteq U\). Therefore \(A \cup B\) is (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\).

Remark 3.1. The intersection of two (1,2)*-rgα-closed sets in \((X, \tau_1, \tau_2)\) is generally not (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\).
Example 3.5. Let $X=\{a, b, c, d, e\}$ and $\tau_1=\{\phi, \{a, b\}, \{a, b, c, d\}, X\}$, $\tau_2=\{\phi, \{c, d\}, \{a, b, c, d\}\}$, then the set $A=\{a, b, c\}$ and $B=\{a, d, e\}$ then $A$ and $B$ are $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$ but $A \cap B=\{a\}$ is not $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$.

**Theorem 3.7.** If a subset $A$ of $(X, \tau_1, \tau_2)$ is $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$. Then $(1,2)^*\alpha$cl$(A)\setminus A$ does not contain any nonempty $(1,2)^*\alpha$-regular $\alpha$-open set in $(X, \tau_1, \tau_2)$.

**Proof.** Suppose that $A$ is $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$. We prove the result by contradiction. Let $U$ be a $(1,2)^*\alpha$-regular $\alpha$-open set such that $(1,2)^*\alpha$ cl$(A)\setminus A \cup U$ and $U=\phi$. Now $U \subseteq \alpha$ cl$(A)\setminus A$. Therefore $U \subseteq X \setminus A$ which implies $A \subseteq X \setminus U$. Since $U$ is $(1,2)^*\alpha$-regular $\alpha$-open set, $X \setminus U$ is also $(1,2)^*\alpha$-regular $\alpha$-open in $(X, \tau_1, \tau_2)$. Since $A$ is $(1,2)^*\alpha$-closed set in $X$, by definition we have $\alpha$ cl$(A)\subseteq X \setminus U$. So $U \subseteq X \setminus \alpha$ cl$(A)$. Also $U \subseteq \alpha$ cl$(A)$. Therefore $U \subseteq \alpha \cap (X \setminus \alpha \cap (A))=\phi$. This shows that, $U=\phi$ which is contradiction. Hence $(1,2)^*\alpha$ cl$(A)\setminus A$ does not contains any non-empty $(1,2)^*\alpha$-regular $\alpha$-open set in $(X, \tau_1, \tau_2)$.

**Corollary 3.5.** If a subset $A$ of $(X, \tau_1, \tau_2)$ is $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$, then $(1,2)^*\alpha$cl$(A)\setminus A$ does not contain any $(1,2)^*\alpha$-regular open set in $(X, \tau_1, \tau_2)$, but not conversely.

**Proof.** Follows from theorem 3.20. and the fact that every $(1,2)^*\alpha$-regular open set is $(1,2)^*\alpha$-regular $\alpha$-open.

**Corollary 3.6.** If a subset $A$ of $(X, \tau_1, \tau_2)$ is $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$, then $\alpha$ cl$(A)\setminus A$ does not contain any non-empty $(1,2)^*\alpha$-regular closed set in $(X, \tau_1, \tau_2)$, but not conversely.

**Proof.** Follows from theorem 3.20. and the fact that every regular open set is $(1,2)^*\alpha$-regular $\alpha$-open.

**Theorem 3.8.** For an element $x \in (X, \tau_1, \tau_2)$, the set $(X, \tau_1, \tau_2)\setminus \{x\}$ is $(1,2)^*\alpha$-closed or $(1,2)^*\alpha$-regular $\alpha$-open.

**Proof.** Suppose $(X, \tau_1, \tau_2)\setminus \{x\}$ is not $(1,2)^*\alpha$-regular $\alpha$-open set. Then $(X, \tau_1, \tau_2)$ is the only $(1,2)^*\alpha$-regular $\alpha$-open set containing $(X, \tau_1, \tau_2)\setminus \{x\}$. This implies $\alpha$ cl$(\{X, \tau_1, \tau_2\}\setminus \{x\}) \subseteq (X, \tau_1, \tau_2)$. Hence $(X, \tau_1, \tau_2)\setminus \{x\}$ is $(1,2)^*\alpha$-closed set in $(X, \tau_1, \tau_2)$.

**Theorem 3.9.** If $A$ is $(1,2)^*\alpha$-regular open and $(1,2)^*\alpha$-rg$\alpha$-closed then $A$ is $(1,2)^*\alpha$-regular closed and hence $(1,2)^*\alpha$-clopen.

**Proof.** Suppose $A$ is $(1,2)^*\alpha$-regular open and $(1,2)^*\alpha$-rg$\alpha$-closed. As every $(1,2)^*\alpha$-regular open is $(1,2)^*\alpha$-regular $\alpha$-open and $A \subseteq A$, we have $\alpha$ cl$(A)\subseteq A$. Also $\subseteq \alpha$ cl$(A)$ Therefore $\alpha$ cl$(A)=A$. That is $A$ is $\alpha$-closed. Since $A$ is $(1,2)^*\alpha$-regular open, $A$ is $\alpha$-open. Now cl$(\text{int}(A))=\alpha$ cl$(A)=A$. Therefore $A$ is a $(1,2)^*\alpha$-regular closed and $\alpha$-clopen.
Theorem 3.10. If \( A \) is \((1,2)^*\)-rg\(\alpha\)-closed subset of \((X, \tau_1, \tau_2)\) such that \( A \subset B \subset (1,2)^*\)-\(\alpha\) cl\(A\). Then \( B \) is \((1,2)^*\)-rg\(\alpha\)-closed subset in \((X, \tau_1, \tau_2)\).

Proof. If \( A \) is \((1,2)^*\)-rg\(\alpha\)-closed subset of \((X, \tau_1, \tau_2)\) such that \( A \subset B \subset (1,2)^*\)-\(\alpha\) cl\(A\). Let \( U \) be a \((1,2)^*\)-regular \(\alpha\)-open set of \((X, \tau_1, \tau_2)\) such that \( B \subset U \). Then \( A \subset U \). Since \( A \) is \((1,2)^*\)-rg\(\alpha\)-closed we have \((1,2)^*\)-regular \(\alpha\)-closed. Hence \( B \) is \((1,2)^*\)-rg\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\).

\(\square\)

Remark 3.2. The converse of the theorem 2.10. need not be true in general. Consider the bitopological space \((X, \tau_1, \tau_2)\) where \( X = \{a, b, c, d, e\} \) with topology \( \tau_1 = \{\phi, \{a, b\}, a, b, c, d, X\} \), \( \tau_2 = \{\phi, \{c, d\}, \{a, b, c, d\}, X\} \). Let \( A = \{b\} \) and \( B = \{b, c\} \). Then \( A \) and \( B \) are \((1,2)^*\)-rg\(\alpha\)-closed sets in \((X, \tau_1, \tau_2)\) but \( A \subset B \) is not subset in \((1,2)^*\)-\(\alpha\) cl\(A\).

Theorem 3.11. Let \( A \) be a \((1,2)^*\)-rg\(\alpha\)-closed in \((X, \tau_1, \tau_2)\). Then \( A \) is \((1,2)^*\)-\(\alpha\)-closed if and only if \((1,2)^*\)-\(\alpha\) cl\(A\) \(\setminus\) \(A\) is a \((1,2)^*\)-regular \(\alpha\)-open.

Proof. Suppose \( A \) is a \((1,2)^*\)-\(\alpha\)-closed in \((X, \tau_1, \tau_2)\). Then \((1,2)^*\)-\(\alpha\) cl\(A\) = \(\phi\) and so \((1,2)^*\)-\(\alpha\) cl\(A\) \(\setminus\) \(A\) = \(\phi\), which is \((1,2)^*\)-regular \(\alpha\)-open in \((X, \tau_1, \tau_2)\). Conversely, suppose \((1,2)^*\)-\(\alpha\) cl\(A\) \(\setminus\) \(A\) is \(\alpha\)rg\((\alpha\)cl\(A\)) \(\alpha\)-regular \(\alpha\)-open set in \((X, \tau_1, \tau_2)\). Since \( A \) is \((1,2)^*\)-rg\(\alpha\)-closed, by theorem 2.7 \((1,2)^*\)-\(\alpha\)cl\(A\) \(\setminus\) \(A\) does not contain any nonempty \((1,2)^*\)-regular \(\alpha\)-open in \((X, \tau_1, \tau_2)\). Then \((1,2)^*\)-\(\alpha\) cl\(A\) \(\setminus\) \(A\) = \(\phi\), hence \( A \) is \((1,2)^*\)-\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\).

\(\square\)

Theorem 3.12. If \( A \) is \((1,2)^*\)-regular open and rg-closed, then \( A \) is \((1,2)^*\)-rg\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\).

Proof. Let \( U \) be any \((1,2)^*\)-regular \(\alpha\)-open set in \((X, \tau_1, \tau_2)\) such that \( A \subset U \). Since \( A \) is \((1,2)^*\)-regular open and \((1,2)^*\)-rg-closed, we have \((1,2)^*\)-\(\alpha\) cl\(A\) \(\subset\) \( A \). Then \((1,2)^*\)-\(\alpha\) cl\(A\) \(\subset\) \( A \subset U \). Hence \( A \) is \((1,2)^*\)-rg\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\).

\(\square\)

Theorem 3.13. If a subset \( A \) of bitopological space \((X, \tau_1, \tau_2)\) is both \((1,2)^*\)-regular \(\alpha\)-open and \((1,2)^*\)-rg\(\alpha\)-closed, then it is \((1,2)^*\)-\(\alpha\)-closed.

Proof. Suppose a subset \( A \) of bitopological space \((X, \tau_1, \tau_2)\) is both \((1,2)^*\)-regular \(\alpha\)-open and \((1,2)^*\)-rg\(\alpha\)-closed. Now \( A \subset A \). Then \((1,2)^*\)-\(\alpha\) cl\(A\) \(\subset\) \( A \). Hence \( A \) is \((1,2)^*\)-\(\alpha\)-closed.

\(\square\)

Corollary 3.7. Let \( A \) be \((1,2)^*\)-regular \(\alpha\)-open and \((1,2)^*\)-rg\(\alpha\)-closed subset in \((X, \tau_1, \tau_2)\). Suppose that \( F \) is \((1,2)^*\)-\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\). Then \( A \cap F \) is an \((1,2)^*\)-rg\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\).

Proof. Let \( A \) be a \((1,2)^*\)-regular \(\alpha\)-open and \((1,2)^*\)-rg\(\alpha\)-closed subset in \((X, \tau_1, \tau_2)\) and \( F \) be closed. By theorem 2.13, \( A \) is \((1,2)^*\)-\(\alpha\)-closed. So \( A \cap F \) is a \((1,2)^*\)-\(\alpha\)-closed and hence \( A \cap F \) is \((1,2)^*\)-rg\(\alpha\)-closed set in \((X, \tau_1, \tau_2)\).

\(\square\)
Theorem 3.14. If $A$ is an open and $S$ is $(1,2)^*$- $\alpha$-open in bitopological space $(X, \tau_1, \tau_2)$, then $A \cap S$ is $(1,2)^*$- $\alpha$-open in $(X, \tau_1, \tau_2)$.

Theorem 3.15. If $A$ is both open and $(1,2)^*$- $g$-closed set in $(X, \tau_1, \tau_2)$, then it is $(1,2)^*$- rgo-closed set in $(X, \tau_1, \tau_2)$.

Proof. Let $A$ be open and $(1,2)^*$- $g$-closed set in $(X, \tau_1, \tau_2)$. Let $A \subset U$ and let $U$ be a $(1,2)^*$-regular $\alpha$-open set in $(X, \tau_1, \tau_2)$. Now $A \subset A$. By hypothesis $(1,2)^*$ $\alpha$-cl($A$) $\subset A$. That is $(1,2)^*$ $\alpha$-cl($A$) $\subset U$. Thus $A$ is $(1,2)^*$-rgo-closed in $(X, \tau_1, \tau_2)$. □

Remark 3.3. If $A$ is both open and $(1,2)^*$- rgo-closed in $X$, then $A$ need not be $(1,2)^*$-g-closed, in general.

Theorem 3.16. In a bitopological space $(X, \tau_1, \tau_2)$, if $R \alpha$ O($(X, \tau_1, \tau_2)) = \{(X, \tau_1, \tau_2), \phi\}$, then every subset of $(X, \tau_1, \tau_2)$ is a $(1,2)^*$- rgo-closed set.

Proof. Let $(X, \tau_1, \tau_2)$ be a bitopological space and $R \alpha$ O($(X, \tau_1, \tau_2)) = \{(X, \tau_1, \tau_2), \phi\}$. Let $A$ be any subset of $(X, \tau_1, \tau_2)$. Suppose $A = \phi$. Then $\phi$ is $(1,2)^*$- rgo-closed set in $(X, \tau_1, \tau_2)$. Suppose $A = \phi$. Then $(X, \tau_1, \tau_2)$ is the only regular $(1,2)^*$- $\alpha$-open set containing $A$ and so $(1,2)^*$ $\alpha$-cl($A$) $\subset (X, \tau_1, \tau_2)$ hence $A$ is $(1,2)^*$-rgo-closed set in $(X, \tau_1, \tau_2)$. □

Theorem 3.17. In a bitopological space $(X, \tau_1, \tau_2)$, $R \alpha$ O $(X, \tau_1, \tau_2) \subset \{F \subset X : F^c \subset \tau_1, \tau_2\}$ iff every subset of $(X, \tau_1, \tau_2)$ is a $(1,2)^*$- rgo-closed set.

Proof. Suppose that $(1,2)^*$ $R \alpha$ O $(X, \tau_1, \tau_2) \subset \{F \subset X : F^c \subset \tau_1, \tau_2\}$. Let $A$ be any subset of $(X, \tau_1, \tau_2)$ such that $A \subset U$, where $U$ is a $(1,2)^*$- regular $\alpha$-open. Then $U \subset (1,2)^*$ $R \alpha$ O $(X, \tau_1, \tau_2) \subset \{F \subset X : F^c \subset \tau_1, \tau_2\}$. That is $U \subset \{F \subset X : F^c \subset \tau\}$. Thus $U$ is a $(1,2)^*$-$\alpha$-closed set. Then $\alpha$ cl($U$) $= U$. Also $(1,2)^*$ $\alpha$-cl($A$) $\subset (1,2)^*$ $\alpha$-cl($U$) $= U$. Hence $A$ is $(1,2)^*$-rgo-closed set in $(X, \tau_1, \tau_2)$. Conversely, suppose that every subset of $(X, \tau_1, \tau_2)$ is $(1,2)^*$-rgo-closed. Let $U \subset (1,2)^*$ $R \alpha$ O $(X, \tau_1, \tau_2)$. Since $U \subset U$ and $U$ is $(1,2)^*$-rgo-closed, we have $(1,2)^*$-$\alpha$ cl($U$) $\subset U$. Thus $(1,2)^*$-$\alpha$ cl($U$) $= U$ and $U \subset \{F \subset X : F^c \subset \tau_1, \tau_2\}$. Therefore $R \alpha$ O $(1,2)^*$ $(X, \tau_1, \tau_2) \subset \{F \subset X : F^c \subset \tau_1, \tau_2\}$. □

Definition 3.3. The intersection of all $(1,2)^*$- regular $\alpha$-open subsets of $(X, \tau_1, \tau_2)$ containing $A$ is called the $(1,2)^*$- regular $\alpha$-kernal of $A$ and is denoted by roker ($A$).

Lemma 3.1. Let $(X, \tau_1, \tau_2)$ be a bitopological space and $A$ be a subset of $(X, \tau_1, \tau_2)$. If $A$ is a $(1,2)^*$- regular $\alpha$-open in $(X, \tau_1, \tau_2)$, then roker ($A$) = $A$ but not conversely.

Proof. Follows from definition. 2.3. □

Lemma 3.2. For any subset $A$ of $(X, \tau_1, \tau_2)$, $\alpha$ker ($A$) $\subset$ roker ($A$).

Proof. Follows from the implication $R \alpha$ O $(X, \tau_1, \tau_2) \subset \alpha$O $(X, \tau_1, \tau_2)$. □
4 (1,2)*-rgα open sets and (1,2)*rgα neighborhood

In this section, we introduce and study (1,2)*-rgα-open sets in topological spaces and obtain some of their properties. Also, we introduce (1,2)*-rgα-neighborhood (shortly (1,2)*-rgα-nbhd in topological spaces by using the notion of (1,2)*-rgα-open sets. We prove that every nbhd of x in \((X, \tau_1, \tau_2)\) is (1,2)*-rgα-nbhd of x but not conversely.

**Definition 4.1.** A subset \(A\) in \((X, \tau_1, \tau_2)\) is called (1,2)*-regular generalized α-open (briefly, (1,2)*-rgα-open) in \((X, \tau_1, \tau_2)\) if \(A^c\) is (1,2)*-rgα-closed in \((X, \tau_1, \tau_2)\). We denote the family of all (1,2)*-rgα-open sets in \(X\) by \((1,2)*-RG\alpha O (X, \tau_1, \tau_2)\).

**Theorem 4.1.** If a subset \(A\) of a space \((X, \tau_1, \tau_2)\) is (1,2)*-w-open then it is (1,2)*-rgα-open but not conversely.

**Proof.** Let \(A\) be a (1,2)*-w-open set in a space \((X, \tau_1, \tau_2)\). Then \(A^c\) is (1,2)*-w-closed set. By theorem 3.2. \(A\) is (1,2)*-rgα-closed. Therefore \(A\) is (1,2)*-rgα-open set in \((X, \tau_1, \tau_2)\). The converse of the above theorem need not be true, as seen from the following example.

**Example 4.1.** Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}\) and \(\tau_2 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}\). In this topological space the subset \(\{a, b\}\) is rg-open set in \(X\), but not (1,2)*-rgα.

**Corollary 4.1.** Every open set is (1,2)*-rgα-open set but not conversely.

**Proof.** Follows from theorem 4.2.

**Corollary 4.2.** Every (1,2)*-regular open set is (1,2)*-rgα-open set but not conversely.

**Proof.** Follows from theorem 4.2.

**Theorem 4.2.** If a subset \(A\) of a space \((X, \tau_1, \tau_2)\) is (1,2)*-rgα-open, then it is (1,2)*-rg-open set in \((X, \tau_1, \tau_2)\).

**Proof.** Let \(A\) be (1,2)*-rgα-open set in space \((X, \tau_1, \tau_2)\). Then \(A^c\) is (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\). By theorem 2.4., \(A^c\) is (1,2)*-rg-closed set in \((X, \tau_1, \tau_2)\). Therefore \(A\) is (1,2)*-rg-open set in space \((X, \tau_1, \tau_2)\). The converse of the above theorem need not be true, as seen from the following example.

**Example 4.2.** Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, \{a\}, X\}\) and \(\tau_2 = \{\phi, \{a\}, \{b, c\}, X\}\). Then the subsets \(A = \{b, c\}\) are (1,2)*-rg-open but not (1,2)*-rgα-open sets in \((X, \tau_1, \tau_2)\).

**Theorem 4.3.** If a subset \(A\) of a space \((X, \tau_1, \tau_2)\) is (1,2)*-rgα-open, then it is (1,2)*-gpr-open set in \((X, \tau_1, \tau_2)\).

**Proof.** Let \(A\) be (1,2)*-rgα-open set in a space \((X, \tau_1, \tau_2)\). Then \(A^c\) is (1,2)*-rgα-closed set in \((X, \tau_1, \tau_2)\). By corollary 2.3. \(A^c\) is (1,2)*-gpr-closed in \((X, \tau_1, \tau_2)\). Therefore \(A\) is (1,2)*-gpr-open set in \((X, \tau_1, \tau_2)\).
Theorem 4.4. If a subset $A$ of a bitopological space $(X,\tau_1,\tau_2)$ is $(1,2)^*-rg\alpha$-open, then it is $(1,2)^*-rwg$-open set in $(X,\tau_1,\tau_2)$, but not conversely.

Proof. Let $A$ be $(1,2)^*-rg\alpha$-open set in a space $(X,\tau_1,\tau_2)$. Then $A^c$ is $(1,2)^*-rg\alpha$-closed set in $(X,\tau_1,\tau_2)$. By theorem 2.5. $A^c$ is $(1,2)^*-rwg$-closed in $(X,\tau_1,\tau_2)$. Therefore $A$ is $(1,2)^*-rwg$-open subset in $(X,\tau_1,\tau_2)$. The converse of the above theorem need not be true, as seen from the following example.

Example 4.3. Let $X = \{a, b, c, d, e\}$ with topology $\tau_1 = \{\emptyset, \{a, b\}, a, b, c, d, X\}$, $\tau_2 = \{\emptyset, \{c, d\}, \{a, b, c, d\}, X\}$. In this topological space the subset $\{a, b\}$ is $(1,2)^*$-rwg-open in $X$, but not $(1,2)^*$-rg\alpha-open.

Theorem 4.5. If $A$ and $B$ are $(1,2)^*$-rg\alpha-open sets in a space $(X,\tau_1,\tau_2)$. Then $A \cap B$ is $(1,2)^*$-rg\alpha-open set in $(X,\tau_1,\tau_2)$.

Proof. If $A$ and $B$ are $(1,2)^*-rg\alpha$-open sets in a space $(X,\tau_1,\tau_2)$. Then $A^c$ and $B^c$ are $(1,2)^*-rg\alpha$-closed sets in a space $(X,\tau_1,\tau_2)$. By theorem 2.6. $A^c \cup B^c$ is also $(1,2)^*-rg\alpha$-closed set in $(X,\tau_1,\tau_2)$. That is $A^c \cup B^c = (A \cap B)^c$ is a $(1,2)^*$-rg\alpha-closed set in $(X,\tau_1,\tau_2)$. Therefore $A \cap B$ is $(1,2)^*-rg\alpha$-open set in $(X,\tau_1,\tau_2)$.

Remark 4.1. The union of two $(1,2)^*$-rg\alpha-open sets in $(X,\tau_1,\tau_2)$ is generally not a $(1,2)^*$-rg\alpha-open set in $(X,\tau_1,\tau_2)$.

Example 4.4. Let $X = \{a, b, c, d, e\}$ with topology $\tau_1 = \{\emptyset, \{a, b\}, a, b, c, d, X\}$, $\tau_2 = \{\emptyset, \{c, d\}, \{a, b, c, d\}, X\}$. If $A = \{a\}$ and $B = \{c\}$, then $A$ and $B$ are $(1,2)^*$-rg\alpha-open sets in $X$, but $A \cup B = \{a, c\}$ is not a $(1,2)^*-rg\alpha$-open set in $X$.

Theorem 4.6. If a set $A$ is $(1,2)^*$-rg\alpha-open in a space $(X,\tau_1,\tau_2)$, then $G = X$, whenever $G$ is $(1,2)^*$-regular $\alpha$-open and $int(A) \cup A^c \subset G$.

Proof. Suppose that $A$ is $(1,2)^*$-rg\alpha-open in $(X,\tau_1,\tau_2)$. Let $G$ be $(1,2)^*$-regular $\alpha$-open and $int(A) \cup A^c \subset G$. This implies $G^c \subset (int(A) \cup A^c)^c = (int(A))^c \cap A$. That is $G^c \subset (int(A))^c - A^c$, Thus $G^c \subset cl(A)^c - A^c$, Since $(int(A))^c = cl(A^c)$. Now $G^c$ is also regular $\alpha$-open and $A^c$ is $(1,2)^*$-rg\alpha-closed, by theorem 2.7., it follows that $G^c = \emptyset$. Hence $G = (X,\tau_1,\tau_2)$. The converse of the above theorem is not true in general as seen from the following example.

Example 4.5. Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a, b, d\}, X\}$. Then $(1,2)^*$-rg\alpha O $(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$ and $(1,2)^*$-rg\alpha O $(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, c, d\}\}$. Take $A = \{b, c, d\}$. Then $A$ is not $(1,2)^*$-rg\alpha-open. However $int(A) \cup A^c = \{b, c\} \cup \{a\} = \{a, b, c\}$. So for some regular $\alpha$-open $G$, we have $int(A) \cup A^c = \{a, b, c\} \subset G$ gives $G = X$, but $A$ is not $(1,2)^*$-rg\alpha-open.
Theorem 4.7. Every singleton point set in a space is either $(1,2)^*$-rg$\alpha$-open or $(1,2)^*$-ra$\alpha$-open.

Proof. Let $(X, \tau_1, \tau_2)$ be a bitopological space. Let $x \in (X, \tau_1, \tau_2)$. To prove $x$ is either $(1,2)^*$-rg$\alpha$-open or $(1,2)^*$-ra$\alpha$-open. That is to prove $(X, \tau_1, \tau_2)\{-x\}$ is either $(1,2)^*$-rg$\alpha$-closed or $(1,2)^*$-ra$\alpha$-open, which follows from theorem 2.8. Analogous to a neighbourhood in space $(X, \tau_1, \tau_2)$, we define $(1,2)^*$-rg$\alpha$-neighbourhood in a space $(X, \tau_1, \tau_2)$ as follows. □

Definition 4.2. Let $(X, \tau_1, \tau_2)$ be a bitopological space and let $x \subset (X, \tau_1, \tau_2)$. A subset $N$ of $(X, \tau_1, \tau_2)$ is said to be a $(1,2)^*$-rg$\alpha$-nbhd of $x$ iff there exists a $(1,2)^*$-rg$\alpha$-open set $G$ such that $x \in G \subset N$.

Definition 4.3. A subset $N$ of space $(X, \tau_1, \tau_2)$, is called a $(1,2)^*$-rg$\alpha$-nbhd of $A \subset (X, \tau_1, \tau_2)$ iff there exists a $(1,2)^*$-rg$\alpha$-open set $G$ such that $A \subset G \subset N$.

Remark 4.2. The $(1,2)^*$-rg$\alpha$-nbhd $N$ of $x \in (X, \tau_1, \tau_2)$ need not be a $(1,2)^*$-rg$\alpha$-open in $(X, \tau_1, \tau_2)$.

Example 4.6. Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b, d\}, X\}$ with topology Then $(1,2)^*$-RG$\alpha$ $O(\{a\}) = \{X, \phi, \{a\}, \{b\}, \{a, b, c\}, \{a, b, c\}, \{a, b, d\}\}$. Note that $\{a, c\}$ is not a $(1,2)^*$-rg$\alpha$-open set, but it is a $(1,2)^*$-rg$\alpha$-nbhd of $\{a\}$. Since $\{a\}$ is a $(1,2)^*$-rg$\alpha$-open set such that $a \in \{a\} \subset \{a, c\}$.

Theorem 4.8. Every nbhd $N$ of $x \in (X, \tau_1, \tau_2)$ is a $(1,2)^*$-rg$\alpha$-nbhd of $(X, \tau_1, \tau_2)$.

Proof. Let $N$ be a nbhd of point $x \in (X, \tau_1, \tau_2)$. To prove that $N$ is a $(1,2)^*$-rg$\alpha$-nbhd of $x$. By definition of nbhd, there exists an open set $G$ such that $x \in G \subset N$. As every open set is $(1,2)^*$-rg$\alpha$-open set $G$ such that $x \in G \subset N$. Hence $N$ is $(1,2)^*$-rg$\alpha$-nbhd of $x$. □

Remark 4.3. In general, a $(1,2)^*$-rg$\alpha$-nbhd $N$ of $x \in (X, \tau_1, \tau_2)$ need not be a nbhd of $x$ in $(X, \tau_1, \tau_2)$, as seen from the following example.

Example 4.7. Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b, d\}, X\}$ Then $(1,2)^*$-RG$\alpha$ $O(\{a, c\}) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. The set $\{a, c\}$ is $(1,2)^*$-rg$\alpha$-nbhd of the point $c$, since the $(1,2)^*$-rg$\alpha$-open sets $\{c\}$ is such that $c \in \{c\} \subset \{a, c\}$. However, the set $\{a, c\}$ is not a nbhd of the point $c$, since no open set $G$ exists such that $c \in G \subset \{a, c\}$.

Theorem 4.9. If a subset $N$ of a space $(X, \tau_1, \tau_2)$ is $(1,2)^*$-rg$\alpha$-open, then $N$ is $(1,2)^*$-rg$\alpha$-nbhd of each of its points.

Proof. Suppose $N$ is $(1,2)^*$-rg$\alpha$-open. Let $x \in N$. We claim that $N$ is $(1,2)^*$-rg$\alpha$-nbhd of $x$. For $N$ is a $(1,2)^*$-rg$\alpha$-open set such that $x \in N \subset N$. Since $x$ is an arbitrary point of $N$, it follows that $N$ is a $(1,2)^*$-rg$\alpha$-nbhd of each of its points. □

Remark 4.4. The converse of the above theorem is not true in general as seen from the following example.
Example 4.8. Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a, b, c\}, X\}$. Then $(1,2)^*-RG\alpha O (X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. The set $\{a, d\}$ is a $(1,2)^*$-rgo-nbd of the point $a$, since the $(1,2)^*$-rgo-open set $\{a\}$ is such that $a \in \{a\} \subset \{a, d\}$. Also the set $\{a, d\}$ is a $(1,2)^*$-rgo-nbd of the point $\{d\}$, since the $(1,2)^*$-rgo-open set $\{d\}$ is such that $d \in \{d\} \subset \{a, d\}$. That is $\{a, d\}$ is a $(1,2)^*$-rgo-nbd of each of its points. However the set $\{a, d\}$ is not a $(1,2)^*$-rgo-open set in $X$.

Theorem 4.10. Let $(X, \tau_1, \tau_2)$ be a bitopological space. If $F$ is a $(1,2)^*$-rgo-closed subset of $(X, \tau_1, \tau_2)$, and $x \in F^c$. Prove that there exists a $(1,2)^*$-rgo-nbd $N$ of $x$ such that $N \cap F = \emptyset$.

Proof. Let $F$ be a $(1,2)^*$-rgo-closed subset of $(X, \tau_1, \tau_2)$ and $x \in F^c$. Then $F^c$ is a $(1,2)^*$-rgo-open set of $(X, \tau_1, \tau_2)$. So by theorem 3.9. $F^c$ contains a $(1,2)^*$-rgo-nbd of each of its points. Hence there exists a $(1,2)^*$-rgo-nbd $N$ of $x$ such that $N \subset F^c$. That is $N \cap F = \emptyset$. \hfill $\blacksquare$

Definition 4.4. Let $x$ be a point in a space $(X, \tau_1, \tau_2)$. The set of all $(1,2)^*$-rgo-nbd of $x$ is called the $(1,2)^*$-rgo-nbd system at $x$, and is denoted by $(1,2)^*$-rgo-N$(x)$.

Theorem 4.11. Let $(X, \tau_1, \tau_2)$ be a bitopological space and for each $x \subset (X, \tau_1, \tau_2)$, Let $(1,2)^*$-rgo-N$(x)$ be the collection of all $(1,2)^*$-rgo-nbhs of $x$. Then we have the following results.

(i) $\forall x \in (X, \tau_1, \tau_2), (1,2)^*$-rgo-N$(x) \neq \emptyset$.

(ii) $N \in (1,2)^*$-rgo-N$(x)$ $\Rightarrow$ $x \in N$.

(iii) $N \in (1,2)^*$-rgo-N$(x)$, $M \supseteq N \Rightarrow M \in (1,2)^*$-rgo-N$(x)$.

(iv) $N \in (1,2)^*$-rgo-N$(x)$, $M \in (1,2)^*$-rgo-N$(x)$ $\Rightarrow N \cap M \in (1,2)^*$-rgo-N$(x)$.

(v) $N \in (1,2)^*$-rgo-N$(x)$ $\Rightarrow$ there exists $M \in (1,2)^*$-rgo-N$(x)$ such that $M \subset N$ and $M \in (1,2)^*$-rgo-N$(y)$ for every $y \in M$.

Proof. (i) Since $(X, \tau_1, \tau_2)$ is a $(1,2)^*$-rgo-open set, it is a $(1,2)^*$-rgo-nbd of every $x \in (X, \tau_1, \tau_2)$. Hence there exists at least one $(1,2)^*$-rgo-nbd (namely - $(X, \tau_1, \tau_2)$) for each $x \in (X, \tau_1, \tau_2)$. Hence $(1,2)^*$-rgo-

(i) $N(x) \neq \emptyset$ for every $x \in (X, \tau_1, \tau_2)$.

(ii) If $N \in (1,2)^*$-rgo-N$(x)$, then $N$ is a $(1,2)^*$-rgo-nbd of $x$. So by definition of $(1,2)^*$-rgo-nbd, $x \in N$.

(iii) Let $N (1,2)^*$-rgo-N$(x)$ and $M \supseteq N$. Then there is a $(1,2)^*$-rgo-open set $G$ such that $x \in G \subset N$. Since $N \subset M, x \in G \subset M$ and so $M$ is $(1,2)^*$-rgo-nbd of $x$. Hence $M \in (1,2)^*$-rgo-N$(x)$.
(iv) Let $N \in (1,2)^*-rg_\alpha - N (x)$ and $M \in (1,2)^*-rg_\alpha - N (x)$. Then by definition of $(1,2)^*-rg_\alpha$-nbhd Hence $x \in G_1 \cap G_2 \subset N \cap M \rightarrow (1)$. Since $G_1 \cap G_2$ is a $(1,2)^*-rg_\alpha$-open set, (being the intersection of two $(1,2)^*-rg_\alpha$-open sets), it follows from (1) that $N \cap M$ is a $(1,2)^*-rg_\alpha$-nbhd of $x$. Hence $N \cap M \in (1,2)^*-rg_\alpha - N (x)$.

(v) If $N \in (1,2)^*-rg_\alpha - N (x)$, then there exists a $(1,2)^*-rg_\alpha$-open set $M$ such that $x \in M \subset N$. Since $M$ is a $(1,2)^*-rg_\alpha$-open set, it is $(1,2)^*-rg_\alpha$-nbhd of each of its points. Therefore $M \in (1,2)^*-rg_\alpha - N (y)$ for every $y \in M$.

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