A solution for fuzzy game matrix using defuzzification method

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Abstract

In this paper to fuzzy game concept is used and the fuzzy game matrix is solved using defuzzification method.

Keywords: Fuzzy numbers, defuzzification, fuzzy matrix game.

2010 MSC: 54A10, 54A20.

1 Introduction

1.1 Fuzzy Logic

Fuzzy logic is a form in which it deals with an unclear situation. In general it is not connected with fixed or exact value but it deals with approximate value. In this, the situation lies possibly between 0 and 1. This is known as membership function. This came into existence by the extension of Boolean logic in which the situation is either true or false i.e 1 and 0 respectively.

Fuzzy grants the association of uncertainty on parameters, properties, geometry, etc., On many aspects fuzzy logic portraits the physical world in a realistic manner than the original numbers (single-valued numbers).

The best example is that in Old AC’s used simple on-off mechanism. When the temperature dropped below a preset level, the AC was turned off. When it rose above a preset level, the AC was turned on. In this case there was a gap between two values. But, by using fuzzy logic when the air is getting warmer, the AC turns the cooling power up and down when it is needed. Hence, this makes the machine smoother and more comfortable.

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1.2 Fuzzy Numbers

A fuzzy set A defined on the set of real numbers R is said to fuzzy number if its membership function $\mu_A : R \rightarrow [0, 1]$ has the following characteristics

(i) A is normal i.e there exist an $x \in R$ such that $\mu_A(x) = 1$

(ii) A is convex i.e for every $x_1, x_2 \in R$, $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$ $\lambda \in [0, 1]$

(iii) $\mu_A$ is upper semi-continuous.

(iv) Supp(A) is bounded in R.

2 Fuzzy Matrix Game

Let $A_i (i=1,2,..,m)$ be the strategies of player A and $B_j (j=1,2,...,n)$ be the strategies of player B where each player has his choice from amongst the pure strategies. It is assumed that player A is always the gainer and player B is always the loser i.e., all payoff’s are assumed in terms of player A. Let $a_{ij}$ be the payoff which player A gains from player B. Then the payoff matrix is of the form

<table>
<thead>
<tr>
<th>PLAYER A</th>
<th>B_1</th>
<th>...</th>
<th>B_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>$(\alpha_{11}, \beta_{11}, \gamma_{11})$</td>
<td>...</td>
<td>$(\alpha_{1n}, \beta_{1n}, \gamma_{1n})$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>A_m</td>
<td>$(\alpha_{m1}, \beta_{m1}, \gamma_{m1})$</td>
<td>...</td>
<td>$(\alpha_{mn}, \beta_{mn}, \gamma_{mn})$</td>
</tr>
</tbody>
</table>

3 Defuzzification method

It is a process in which we get a quantifiable result using membership degrees. For example, rules designed to select how much pressure to apply may result in “decrease pressure (25), maintain pressure (40% percent) and increase pressure (76)”. The simplest defuzzification method is done by choosing the highest value ignoring all other values. There are so many methods of defuzzification methods such as centre of area, centre of gravity, fuzzy mean, random choice of maximum, etc.
4  Basic Definitions

4.1  Pure Strategy

It is one of the decision making process where one action is choosen. Here we use min-max principle to solve the fuzzy matrix game using defuzzification method which is defined as

\[\max - \min = \hat{j}(\alpha_{ij}, \beta_{ij}, \gamma_{ij}) = \hat{i}(\alpha_{ij}, \beta_{ij}, \gamma_{ij})\]

4.2  Saddle Point

Neumann introduced the concept of saddle point. The (k,r)th position of the pay off fuzzy matrix game is known to be the saddle point iff

\[(\alpha_{kr}, \beta_{kr}, \gamma_{kr}) = \hat{i}(\alpha_{ij}, \beta_{ij}, \gamma_{ij}) = \hat{j}(\alpha_{ij}, \beta_{ij}, \gamma_{ij})\]

This value is said to be the value of the game.

4.3  Expected pay-off

Let \(x = (x_1, x_2, \ldots, x_m)\) and \(y = (y_1, y_2, \ldots, y_m)\) be the mixed strategies of player A and player B respectively. The expected pay-off is defined to be

\[E(x, y) = \sum_{i=1}^{n} \sum_{j=1}^{n} (\alpha_{ij}, \beta_{ij}, \gamma_{ij}) x_i y_j\]

5  Numerical Example

Consider the following fuzzy matrix game problem

**Solution:**

The given problem is converted using defuzzification method.

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(B_1)</td>
</tr>
<tr>
<td>(A_1)</td>
<td>(0.62, 0.5, 0.1)</td>
</tr>
<tr>
<td>(A_2)</td>
<td>(0.12, 0.3, 0.4)</td>
</tr>
<tr>
<td>(A_3)</td>
<td>(0.9, 0.15, 0.16)</td>
</tr>
</tbody>
</table>
Therefore, there exists a saddle point. The value of the game = 0.5.

6 Conclusion

In this paper, the fuzzy matrix game is considered and solved using defuzzification method. The pay-off value of the players is in fuzzy numbers and it is defuzzified choosing the highest value. Numerical example is also discussed.

References


Received: May 13, 2015; Accepted: June 19, 2015