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A curious summation formula in the light of Gamma function and contiguous relation

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Abstract

The main aim of the present paper is to establish a curious summation formula involving recurrence relation of Gamma function .

Keywords: Gauss second summation theorem ,Recurrence relation, Prudnikov.

2010 MSC: 33C05 , 33C20 , 33D15 , 33D50 , 33D60.

1 Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{lr} a_1,a_2,\cdots,a_A \;; & \\ & z \\ b_1,b_2,\cdots,b_B \;; & \end{array}\right] = \sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k k!} \tag{1}$$

.

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $|\, z \,|= 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$(a-b)\,{}_2F_1\left[\begin{array}{ll} a,\ b\,; & \\ c\ ; & z\end{array}\right] = a\,{}_2F_1\left[\begin{array}{ll} a+1,\ b\ ; & \\ c\ ; & z\end{array}\right] - b\,{}_2F_1\left[\begin{array}{ll} a,\ b+1\ ; & \\ c\,; & z\end{array}\right] \tag{2}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$${}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ \frac{a+b+1}{2}\,; & \frac{1}{2}\end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\,\Gamma(\frac{b+1}{2})} \tag{3}$$

$$= \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{a+b+1}{2})}{\Gamma(b)\,\Gamma(\frac{a+1}{2})} \tag{4}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$${}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ \frac{a+b-1}{2}\,; & \frac{1}{2}\end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\,\Gamma(\frac{b+1}{2})} + \frac{2\,\Gamma(\frac{a+b-1}{2})}{\Gamma(a)\,\Gamma(b)}\right] \tag{5}$$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \tag{7} $$

2 Main summation formula

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+48}{2} & ; & \frac{1}{2} \end{matrix}\right] = \frac{2^{b}\,\Gamma(\frac{a+b+48}{2})}{(a-b)\,\Gamma(b)\Big[\prod_{\Lambda=1}^{23}\{a-b-2\Lambda\}\Big]\Big[\prod_{\Psi=1}^{23}\{a-b+2\Psi\}\Big]} $$

$$ \Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\Big\{8388608(-216862434431944426122117120000 + a^{23} + 559843369263277204857421824000b $$
$$ -569545783776841112218710835200b^2 + 350200961782994226978068889600b^3 $$
$$ -130274290623536732525341704192b^4 + 38258159328821814810743144448b^5 $$
$$ -7392547502167306440045232128b^6 + 1278328901424788437956820992b^7 $$
$$ -147385136152123509508145152b^8 + 16541594137308004947263488b^9 $$
$$ -1217098252828610199584768b^{10} + 93319246373147360817152b^{11} - 4520782665435130478592b^{12} $$
$$ +242909449904204187648b^{13} - 7797274383016572928b^{14} + 295577121333620992b^{15} $$
$$ -6177274611310592b^{16} + 163279973416448b^{17} - 2112944531328b^{18} + 37538840592b^{19} $$
$$ -268641472b^{20} + 2948968b^{21} - 8648b^{22} + 47b^{23} + 23a^{22}(-24+47b) + 253a^{21}(568 - 752b + 705b^2) $$
$$ +1771a^{20}(-13248 + 28952b - 10152b^2 + 6063b^3) + 253a^{19}(10651312 - 18378880b + 18307440b^2 $$
$$ -2910240b^3 + 1242915b^4) + 4807a^{18}(-48160896 + 111412560b - 59317760b^2 + 36701360b^3 $$
$$ -3314440b^4 + 1077193b^5) + 437a^{17}(35335680512 - 68886941952b + 71068775952b^2 - 18381075840b^3 $$
$$ +8004372600b^4 - 465347376b^5 + 119568423b^6) + 7429a^{16}(-110280732672 + 264410732032b $$
$$ -172039726464b^2 + 108639162192b^3 - 16625231040b^4 + 5446950696b^5 - 220741704b^6 $$
$$ +459987855b^7) + 14858a^{15}(2365174520192 - 4965191440384b + 5236842909696b^2 $$
$$ -1725798706176b^3 + 758786681184b^4 - 76810158336b^5 + 19719592224b^6 - 588644544b^7 + 101173281b^8) $$
$$ +874a^{14}(-1409491500899328 + 3463171135496576b - 2549281676298240b^2 + 1628606520320000b^3 $$
$$ -326241761038080b^4 + 106997067614688b^5 - 7699470635520b^6 + 1590811880160b^7 - 36422381160b^8 $$
$$ +5227286185b^9) + 46a^{13}(772492387032024064 - 1706132584134203392b + 1825151909639889024b^2 $$
$$ -700251511732561920b^3 + 308859506811100160b^4 - 41720602570934784b^5 + 10644260621618400b^6 $$
$$ -572668731075840b^7 + 96968519441640b^8 - 1756368158160b^9 + 212227819111b^{10}) $$
$$ +598a^{12}(-1415643061687443456 + 3539464620490595328b - 2837813317389384704b^2 $$
$$ +1823283877135639168b^3 - 433637769393797120b^4 + 141648230090454016b^5 $$
$$ -13801291695856896b^6 + 2815337632560096b^7 - 117555258659520b^8 + 16518224344600b^9 $$
$$ -242546078984b^{10} + 24805848987b^{11}) + 2a^{11}(833614654875150237900 8 $$

$$
\begin{gathered}
-1911087953173026329395\,2b + 20623641739160852121600b^2 - 8803708000865147375616b^3 \\
+3877040498846657697152b^4 - 630933569726210070528b^5 + 15929039277668986060\,8b^6 \\
-11738492743825376256b^7 + 1950421219324385808b^8 - 65053869300351360b^9 \\
+7649418238997392b^{10} - 92873098607328b^{11} + 8061900920775b^{12}) \\
+22a^{10}(-1231842634233454033305\,6 + 31189294080015037855744b - 26643664822555271921664b^2 \\
+17140623788483946063872b^3 - 4605072082300887901184b^4 + 1492188460507590418560b^5 \\
-177180979433782351872b^6 + 35548416593321988096b^7 - 2052494989469688192b^8 \\
+281304925678327408b^9 - 7668336833158144b^{10} + 758463638626512b^{11} - 7739424883944b^{12} \\
+570534526701b^{13}) + 22a^9(16457315047195645571891\,2 - 388241091264007637073920b \\
+420943316891085447997440b^2 - 194683884826863646556160b^3 + 85278443565011756866560b^4 \\
-15861465348367839070208b^5 + 3948145039255526734976b^6 - 357922456741054965760b^7 \\
+58142553453809144320b^8 - 2703140922913516800b^9 + 308127143096167824b^{10} \\
-6994146932156800b^{11} + 583323319956520b^{12} - 5071418015120b^{13} + 316963625945b^{14}) \\
+22a^8(-1794419261680900851892224 + 4584366935603738318766080b \\
-4114251545048432952852480b^2 + 2640794540104311940581376b^3 - 778226828495277913989120b^4 \\
+249266337772945624175616b^5 - 34151179297906370239488b^6 + 6714896930011659811968b^7 \\
-480769634811446323200b^8 + 64026820139834140160b^9 - 2449850955905578368b^{10} \\
+233377227411710928b^{11} - 4479970542016320b^{12} + 315039984628920b^{13} - 2360832524280b^{14} \\
+124599494337b^{15}) + a^7(347649718060675616799195136 - 838882082577028634028015616b \\
+910822905671321377263845376b^2 - 448673507687430886271483904b^3 \\
+194828024586480427411873792b^4 - 40139945352469260322013184b^5 \\
+9816269459458694551068672b^6 - 1035093445173598250778624b^7 + 163773463835797810765056b^8 \\
-9507758473057537904640b^9 + 1046292644614417620992b^{10} - 33543815778321598464b^{11} \\
+2675398056697299648b^{12} - 44004970534863360b^{13} + 2602500541819200b^{14} - 16977685938048b^{15} \\
+751616304549b^{16}) + a^6(-2436468094409369161115369472 + 6263040594757311926167928832b \\
-5848486988767053581851295744b^2 + 3733541278446785126654017536b^3 \\
-1184188234508519054509670400b^4 + 373651019226936518470082560b^5 \\
-57174017531600230727811072b^6 + 10975982455244081771618304b^7 \\
-920520158677719502915584b^8 + 118626675019529327475968b^9 - 5703526377919620050944b^{10} \\
+520823675966042701824b^{11} - 14201827298789125632b^{12} + 947443730648350656b^{13} \\
-13503082015288320b^{14} + 668046217895616b^{15} - 3826410277704b^{16} + 140676848445b^{17}) \\
+a^5(13311549776672286362560364544 - 32711559967534938612352155648b \\
+35464011035200609536911081472b^2 - 18392078988954334205143154688b^3 \\
+7891021782941046181484429312b^4 - 1764877986073715914181836800b^5 \\
+422468597935738118375137280b^6 - 50111992196873469323575296b^7 \\
+7690160291401572696950784b^8 - 526300747131570952679424b^9 + 55643641282334619534592b^{10} \\
-2254723506020816523264b^{11} + 171002161902726342656b^{12} - 4017063029686619136b^{13} \\
+223281440905959744b^{14} - 2784340493839872b^{15} + 114245678291448b^{16} - 578784747888b^{17}
\end{gathered}
$$

$$
\begin{aligned}
&+17417133617b^{18}) + a^4(-55053024365598449590377381888 + 142044977238750609063203045376b\\
&-137077990719062800861023436800b^2 + 86760711724153184015648555008b^3\\
&-29240401716693618871160012800b^4 + 9055639018840596931534258176b^5\\
&-1515481756258497614495416320b^6 + 282841276651081614820990976b^7\\
&-26859572569417039346319360b^8 + 3332660266936043603056640b^9\\
&-190033682028660149368832b^{10} + 16535976570423823189248b^{11} - 573130879234289725440b^{12}\\
&+36015944570559027200b^{13} - 737043756023662080b^{14} + 33885424088126016b^{15}\\
&-372890329765440b^{16} + 12527300511720b^{17} - 56488000920b^{18} + 1362649145b^{19})\\
&+a^3(165020589921079405558156492800 - 411610098505191809379748282368b\\
&+444320342901862871586922561536b^2 - 240640522263922110933453766656b^3\\
&+101643072715132417602424406016b^4 - 24347580368876167133346660352b^5\\
&+5679836270384155287099801600b^6 - 741920115370433100621545472b^7\\
&+109848794149157432560799744b^8 - 8566222273855029320908800b^9\\
&+864659016160210118774784b^{10} - 41800643016639985786880b^{11} + 2991812117724256942336b^{12}\\
&-89845526485916487680b^{13} + 4650290892979978240b^{14} - 83728552007841792b^{15}\\
&+3147554863570800b^{16} - 30798996193920b^{17} + 831387500720b^{18} - 3354213280b^{19} + 62891499b^{20})\\
&+a^2(-334579316086154168723570688000 + 864548080425026367554322432000b\\
&-858014569647562727164653600768b^2 + 536543437730683821982167859200b^3\\
&-190477895841886852481443430400b^4 + 57637354507041387641790529536b^5\\
&-10406128775890004696560041984b^6 + 1877582322927794908763521024b^7\\
&-197661445831681191815577600b^8 + 23453086420144706587299840b^9\\
&-1533547764773194725654528b^{10} + 126112680824475795320832b^{11} - 5247383586349278087168b^{12}\\
&+307487912205751326976b^{13} - 8093782952810557440b^{14} + 341481867848146944b^{15}\\
&-5458828464945024b^{16} + 164912278071984b^{17} - 1444736263680b^{18} + 30474254800b^{19}\\
&-110443608b^{20} + 1533939b^{21}) + a(404913773986418277702696960000\\
&-1022531955622549936145222860800b + 1095505011472615290568578170880b^2\\
&-616231254291188275591639990272b^3 + 255052972994161204371394658304b^4\\
&-64865795499271462272381222912b^5 + 14657673560084855811375890432b^6\\
&-2080501687036857250211692544b^7 + 294962572320681686052175872b^8\\
&-25677211257278696652472320b^9 + 2451531478102328114786304b^{10}\\
&-136847793704700345286656b^{11} + 9138388985836193208320b^{12} - 331765554383021232128b^{13}\\
&+15765373619992310016b^{14} - 367721195189587968b^{15} + 12439930291235328b^{16}\\
&-178105159009536b^{17} + 4212252152528b^{18} - 33249138560b^{19} + 524821176b^{20} - 1712304b^{21}\\
&+16215b^{22}))\Big\} - \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Big\{8388608(-216862434431944426122117120000 + 47a^{23}\\
&+404913773986418277702696960000b - 334579316086154168723570688000b^2\\
&+165020589921079405558156492800b^3 - 55053024365598449590377381888b^4\\
&+13311549776672286362560364544b^5 - 2436468094409369161115369472b^6
\end{aligned}
$$

$$+3476497180606756167991951 36b^{7}-39477223756979818741628928b^{8}$$
$$+3620609310383042025816064b^{9}-2710053795313598873272 32b^{10}+16672293097503004758016b^{11}$$
$$-846554550889091186688b^{12}+35534649803473106944b^{13}-1231895571786012672b^{14}$$
$$+35141763021012736b^{15}-819275563020288b^{16}+15441692383744b^{17}-231509427072b^{18}$$
$$+2694781936b^{19}-23462208b^{20}+143704b^{21}-552b^{22}+b^{23}+1081a^{22}(-8+15b)$$
$$+11891a^{21}(248-144b+129b^{2})+11891a^{20}(-22592+44136b-9288b^{2}+5289b^{3})$$
$$+11891a^{19}(3156912-2796160b+2562800b^{2}-282080b^{3}+114595b^{4})+20539a^{18}(-102874752$$
$$+205085552b-70341120b^{2}+40478480b^{3}-2750280b^{4}+848003b^{5})+20539a^{17}(7949752832$$
$$-8671559424b+8029226256b^{2}-1499537280b^{3}+609927480b^{4}-28179792b^{5}+6849255b^{6})$$
$$+349163a^{16}(-17691664384+35627859456b-15634040448b^{2}+9014571600b^{3}-1067954880b^{4}$$
$$+327198696b^{5}-10958808b^{6}+2152623b^{7})+41078a^{15}(7195509064064-8951779424256b$$
$$+8313011048448b^{2}-2038282097664b^{3}+824904427872b^{4}-67781793024b^{5}+16262871072b^{6}$$
$$-413303616b^{7}+66731313b^{8})+2162a^{14}(-3606509890386944+7292032201661568b$$
$$-3743655389829120b^{2}+2150920857067520b^{3}-340908305283840b^{4}+103275412074912b^{5}$$
$$-6245643855360b^{6}+1203746781600b^{7}-24023272680b^{8}+3225346795b^{9})$$
$$+2162a^{13}(112354047134229504-153453077882988544b+142223826182123648b^{2}$$
$$-41556672750192640b^{3}+16658623760665600b^{4}-1858031003555328b^{5}+438225592344288b^{6}$$
$$-20353825409280b^{7}+3205772276520b^{8}-51605548720b^{9}+5805624231b^{10})$$
$$+1222a^{12}(-3699494816231694336+7478223392664642560b-4294094587847199744b^{2}$$
$$+2448291422032943488b^{3}-469010539471595520b^{4}+139936302702722048b^{5}$$
$$-11621789933542656b^{6}+2189360111863584b^{7}-80654134144320b^{8}+10501729164520b^{9}$$
$$-139334981544b^{10}+13194600525b^{11})+94a^{11}(992757940139865540608-1455827592603195162624b$$
$$+1341624264090168035328b^{2}-444687691666382827520b^{3}+175914644366210884992b^{4}$$
$$-23986420276817197056b^{5}+5540677403894071296b^{6}-356849104024697856b^{7}$$
$$+54620202160187664b^{8}-1636928005398400b^{9}+177512766487056b^{10}-1976023374624b^{11}$$
$$+157807422279b^{12})+1034a^{10}(-1177077613954168471552+2370920191588325062656b$$
$$-1483121629374462984192b^{2}+836227288356102629376b^{3}-183784992290773838848b^{4}$$
$$+53813966423921295488b^{5}-5515982957369071616b^{6}+1011888437731545088b^{7}$$
$$-52124488423522944b^{8}+6555896661620592b^{9}-163156102833152b^{10}+14795779959376b^{11}$$
$$-140273264248b^{12}+9441469709b^{13})+1034a^{9}(1599767324691296416563 2$$
$$-2483289289872214376448 0b+2268190176029468722176 0b^{2}-8284547653631556403200b^{3}$$
$$+3223075693361744296960b^{4}-508994919856451598336b^{5}+114725991314825268352b^{6}$$
$$-9195124248604968960b^{7}+1362272768932641280b^{8}-57513636657734400b^{9}+5985211184645264b^{10}$$
$$-125829534430080b^{11}+9553092996200b^{12}-78136301040b^{13}+4418421785b^{14})$$
$$+94a^{8}(-156792698034173946285260 8+3137899705539166872895488b$$
$$-2102781338634906295910400b^{2}+1168604193076142899582976b^{3}-285740133717202546237440b^{4}$$
$$+81810215865974177627136b^{5}-9792767645507654286336b^{6}+1742270891870189476224b^{7}$$
$$-112520552828210841600b^{8}+13607831659402140160b^{9}-480371167748224896b^{10}$$

$$+414983238154412464b^{11}-747851539131840b^{12}+47452679726760b^{13}-338650650360b^{14}$$
$$+15991836267b^{15})+47a^{7}(27198487264357200807591936-44265993341209728727908352b$$
$$+39948560062293508697096192b^{2}-15785534369583682991947776b^{3}$$
$$+6017899503214502442999808b^{4}-1066212599933478070714368b^{5}+233531541600937910034432b^{6}$$
$$-22023264790927622356992b^{7}+3143143243835245018368b^{8}-167538171240493813760b^{9}$$
$$+16639684362831568896b^{10}-499510329524484096b^{11}+35820678814275264b^{12}$$
$$-560484289989120b^{13}+29582331558720b^{14}-186086822016b^{15}+7269016485b^{16})$$
$$+47a^{6}(-157288244726963966809473024+311865394895422464071827456b$$
$$-221406995231702227586383872b^{2}+120847580220939474193612800b^{3}$$
$$-32244292686351013074370560b^{4}+8988693573100811029258240b^{5}$$
$$-1216468458119153845272576b^{6}+208856797009759458533376b^{7}-15985658394764683941888b^{8}$$
$$+1848067890715352939776b^{9}-82935777607302377472b^{10}+6778314586242121728b^{11}$$
$$-175599413491966464b^{12}+10417786991371200b^{13}-143177390115840b^{14}+6233908537536b^{15}$$
$$-34891279128b^{16}+1111731933b^{17})+47a^{5}(814003389974932230015811584$$
$$-1380123308495137495157047296b+1226326691639178460463628288b^{2}$$
$$-518033624869705683688226816b^{3}+192673170613629721947537408b^{4}$$
$$-37550595448376934344294400b^{5}+7950021685679500392980480b^{6}-854041390478069368553472b^{7}$$
$$+116677860234144760252416b^{8}-7424515694980690628608b^{9}+698471194280148706560b^{10}$$
$$-26848237009625960448b^{11}+1802247693491308544b^{12}-40832930175808512b^{13}$$
$$+1989690150962496b^{14}-24281815586304b^{15}+860965887672b^{16}-4326740496b^{17}+110171633b^{18})$$
$$+47a^{4}(-2771793417522058138837057536+5426658999875770305774354432b$$
$$-4052721188125252180456243200b^{2}+2162618568407072714945200128b^{3}$$
$$-622136206738162103641702400b^{4}+167894080488107365563498496b^{5}$$
$$-25195494351245086266163200b^{6}+4145277118861285689614336b^{7}-364276387806300300165120b^{8}$$
$$+39917569328303375554560b^{9}-2155565655545096464384b^{10}+164980446759432242432b^{11}$$
$$-5517348640372142080b^{12}+302288027942778880b^{13}-6066708492495360b^{14}+239873457638976b^{15}$$
$$-2627847689280b^{16}+74423634600b^{17}-338989640b^{18}+6690585b^{19})$$
$$+47a^{3}(7451084293255196318682316800-13111303282791239906205106176b$$
$$+11415817824057102595365273600b^{2}-5120011111998342785818165248b^{3}$$
$$+1845972589875599659907416064b^{4}-391320829552219876705173504b^{5}$$
$$+79437048477591172907532288b^{6}-9546244844413423112159232b^{7}$$
$$+1236116593240316227506176b^{8}-91128626940234047324160b^{9}+8023270709503123689472b^{10}$$
$$-374625872377240313856b^{11}+23198377841002387712b^{12}-685352543397826560b^{13}$$
$$+30285151037440000b^{14}-545572705880064b^{15}+17171922040944b^{16}-170904896640b^{17}$$
$$+3753690160b^{18}-15665760b^{19}+228459b^{20})+47a^{2}(-12117995399507257706781081600$$
$$+23308617265374793416352727040b-18255629141437504833290502144b^{2}$$
$$+9453624317060912161423884288b^{3}-2916552994022612784277094400b^{4}$$
$$+754553426280864032700235776b^{5}-124435893378022416635133952b^{6}$$

$$
\begin{aligned}
&+1937921075896428462263500 8b^7 - 1925819872150330318356480b^8 + 1970372972681676565509440b^9 \\
&-1247150268289821238886 4b^{10} + 877601776134504345600b^{11} - 361066460382734479 36b^{12} \\
&+178631889028584883 2b^{13} - 47405791172014080b^{14} + 1655510892601344b^{15} - 27193258040448b^{16} \\
&+660788406192b^{17} - 6066818560b^{18} + 98548560b^{19} - 382536b^{20} + 3795b^{21}) \\
&+47a(1191156104815483414590259200 0 - 2175599905579893481160048640 0b \\
&+1839464000904311420328345600 0b^2 - 875766167032322998680315494 4b^3 \\
&+302223355827128955453623500 8b^4 - 695990637607126353454301184b^5 \\
&+133256182867176849492934656b^6 - 17848554948447417745276928b^7 \\
&+2145873884750686021550080b^8 - 181729872506556766289920b^9 + 14599244037453847506944b^{10} \\
&-813228916243840991232b^{11} + 45034039213901617152b^{12} - 1669831890854752256b^{13} \\
&+64400246221787392b^{14} - 1569634349387776b^{15} + 41793772941824b^{16} - 640501992192b^{17} \\
&+11394897360b^{18} - 98933120b^{19} + 1090936b^{20} - 4048b^{21} + 23b^{22}))\Big\}\Bigg]
\end{aligned}
\tag{8}
$$

3 Derivation of the summation Formula

Putting $c = \frac{a+b+48}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$
(a-b)\,{}_2F_1\left[\begin{array}{ll} a,\ b\ \ ; & \frac{1}{2} \\ \frac{a+b+48}{2}\ ; & \end{array}\right] = a\,{}_2F_1\left[\begin{array}{ll} a+1,\ b\ ; & \frac{1}{2} \\ \frac{a+b+48}{2}\ \ ; & \end{array}\right] - b\,{}_2F_1\left[\begin{array}{ll} a,\ b+1\ ; & \frac{1}{2} \\ \frac{a+b+48}{2}\ \ ; & \end{array}\right]
$$

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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