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Generalization for character formulas in terms of continued fraction identities

M.P. Chaudhary*

International Scientific Research and Welfare Organization, New Delhi, India.

Abstract

In recent work, Folsom discussed character formulas for classical mock theta functions of Ramanujan. Here, we suggest representations for character formulas in terms of continued fraction identities or in more precise language, we can say an applications of continued fraction identities to character formulas. As a consequence, we obtain fourteen new results.

Keywords: Character formulas, q-product identities, continued fractions identities.

2010 MSC: 11F27, 11F37, 11F50, 17B67, 33D15.

1 Introduction and Basic Terminology

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1-aq^{(n-1)}) \tag{1.2}$$

$$(a_1, a_2, a_3, ..., a_k; q)_\infty = (a_1;q)_\infty(a_2;q)_\infty(a_3;q)_\infty...(a_k;q)_\infty \tag{1.3}$$

Ramanujan has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \quad ; \quad |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [1, p.35] is given, as

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\Phi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)_\infty^2(q^2;q^2)_\infty \tag{1.6}$$

$$\Psi(q) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

$$f(-q) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

*Corresponding author.
E-mail address: mpchaudhary_2000@yahoo.com (M.P. Chaudhary)

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

$$(q;q^2)_\infty^{-1} = (-q;q)_\infty \tag{1.9}$$

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (q^a,q^b,q^c \cdots q^t;q^n)_\infty \tag{1.10}$$

$$(q^a;q^n)_\infty(q^a;q^n)_\infty(q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (q^a,q^a,q^c \cdots q^t;q^n)_\infty \tag{1.11}$$

Computation of q-product identities:
In [1], Chaudhary has computed several q-product identities. Here we are giving some identities from [1], and some new identities have been computed, are useful for next section of this paper, as given below

$$(q^2;q^2)_\infty = \prod_{n=0}^{\infty}(1-q^{2n+2})$$

$$= \prod_{n=0}^{\infty}(1-q^{2(4n)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+1)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+2)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+3)+2})$$

$$= \prod_{n=0}^{\infty}(1-q^{8n+2}) \times \prod_{n=0}^{\infty}(1-q^{8n+4}) \times \prod_{n=0}^{\infty}(1-q^{8n+6}) \times \prod_{n=0}^{\infty}(1-q^{8n+8})$$

$$= (q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty(q^8;q^8)_\infty$$

$$= (q^2,q^4,q^6,q^8;q^8)_\infty \tag{1.12}$$

$$(q^4;q^4)_\infty = \prod_{n=0}^{\infty}(1-q^{4n+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{4(3n)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+1)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{12n+4}) \times \prod_{n=0}^{\infty}(1-q^{12n+8}) \times \prod_{n=0}^{\infty}(1-q^{12n+12})$$

$$= (q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty$$

$$= (q^4,q^8,q^{12};q^{12})_\infty \tag{1.13}$$

$$(q^4;q^{12})_\infty = \prod_{n=0}^{\infty}(1-q^{12n+4}) = \prod_{n=0}^{\infty}(1-q^{12(5n)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+1)+4}) \times$$

$$\times \prod_{n=0}^{\infty}(1-q^{12(5n+2)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+3)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{60n+4}) \times \prod_{n=0}^{\infty}(1-q^{60n+16}) \times \prod_{n=0}^{\infty}(1-q^{60n+28}) \times$$

$$\times \prod_{n=0}^{\infty}(1-q^{60n+40}) \times \prod_{n=0}^{\infty}(1-q^{60n+52})$$

$$= (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$

$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \tag{1.14}$$

Similarly we can compute following, as

$$(q^1;q^1)_\infty = (q^1;q^2)_\infty(q^2;q^2)_\infty$$

$$= (q^1,q^2;q^2)_\infty \tag{1.15}$$

$$(q^2;q^2)_\infty = (q^2;q^4)_\infty(q^4;q^4)_\infty$$

$$= (q^2, q^4; q^4)_\infty \tag{1.16}$$

$$\begin{aligned}(q^2; q^2)_\infty &= (q^2; q^8)_\infty (q^4; q^8)_\infty (q^6; q^8)_\infty (q^8; q^8)_\infty \\ &= (q^2, q^4, q^6, q^8; q^8)_\infty\end{aligned} \tag{1.17}$$

$$\begin{aligned}(q^2; q^2)_\infty &= (q^2; q^{12})_\infty (q^4; q^{12})_\infty (q^6; q^{12})_\infty (q^8; q^{12})_\infty (q^{10}; q^{12})_\infty (q^{12}; q^{12})_\infty \\ &= (q^2, q^4, q^6, q^8, q^{10}, q^{12}; q^{12})_\infty\end{aligned} \tag{1.18}$$

$$\begin{aligned}(q^2; q^2)_\infty &= (q^2; q^{16})_\infty (q^4; q^{16})_\infty (q^6; q^{16})_\infty (q^8; q^{16})_\infty (q^{10}; q^{16})_\infty \times \\ &\times (q^{12}; q^{16})_\infty (q^{14}; q^{16})_\infty (q^{16}; q^{16})_\infty \\ &= (q^2, q^4, q^6, q^8, q^{10}, q^{12}, q^{14}, q^{16}; q^{16})_\infty\end{aligned} \tag{1.19}$$

$$\begin{aligned}(q^2; q^2)_\infty &= (q^2; q^{20})_\infty (q^4; q^{20})_\infty (q^6; q^{20})_\infty (q^8; q^{20})_\infty (q^{10}; q^{20})_\infty (q^{12}; q^{20})_\infty \times \\ &\times (q^{14}; q^{20})_\infty (q^{16}; q^{20})_\infty (q^{18}; q^{20})_\infty (q^{20}; q^{20})_\infty \\ &= (q^2, q^4, q^6, q^8, q^{10}, q^{12}, q^{14}, q^{16}, q^{18}, q^{20}; q^{20})_\infty\end{aligned} \tag{1.20}$$

$$\begin{aligned}(q^3; q^3)_\infty &= (q^3; q^6)_\infty (q^6; q^6)_\infty \\ &= (q^3, q^6; q^6)_\infty\end{aligned} \tag{1.21}$$

$$\begin{aligned}(q^4; q^4)_\infty &= (q^4; q^{12})_\infty (q^8; q^{12})_\infty (q^{12}; q^{12})_\infty \\ &= (q^4, q^8, q^{12}; q^{12})_\infty\end{aligned} \tag{1.22}$$

$$\begin{aligned}(q^4; q^4)_\infty &= (q^4; q^{16})_\infty (q^8; q^{16})_\infty (q^{12}; q^{16})_\infty (q^{16}; q^{16})_\infty \\ &= (q^4, q^8, q^{12}, q^{16}; q^{16})_\infty\end{aligned} \tag{1.23}$$

$$\begin{aligned}(q^4; q^4)_\infty &= (q^4; q^{20})_\infty (q^8; q^{20})_\infty (q^{12}; q^{20})_\infty (q^{16}; q^{20})_\infty (q^{20}; q^{20})_\infty \\ &= (q^4, q^8, q^{12}, q^{16}, q^{20}; q^{20})_\infty\end{aligned} \tag{1.24}$$

$$\begin{aligned}(q^4; q^4)_\infty &= (q^4; q^{24})_\infty (q^8; q^{24})_\infty (q^{12}; q^{24})_\infty (q^{16}; q^{24})_\infty (q^{20}; q^{24})_\infty (q^{24}; q^{24})_\infty \\ &= (q^4, q^8, q^{12}, q^{16}, q^{20}, q^{24}; q^{24})_\infty\end{aligned} \tag{1.25}$$

$$\begin{aligned}(q^4; q^{12})_\infty &= (q^4; q^{60})_\infty (q^{16}; q^{60})_\infty (q^{28}; q^{60})_\infty (q^{40}; q^{60})_\infty (q^{52}; q^{60})_\infty \\ &= (q^4, q^{16}, q^{28}, q^{40}, q^{52}; q^{60})_\infty\end{aligned} \tag{1.26}$$

$$\begin{aligned}(q^6; q^6)_\infty &= (q^6; q^{12})_\infty (q^{12}; q^{12})_\infty \\ &= (q^6, q^{12}; q^{12})_\infty\end{aligned} \tag{1.27}$$

$$\begin{aligned}(q^6; q^6)_\infty &= (q^6; q^{24})_\infty (q^{12}; q^{24})_\infty (q^{18}; q^{24})_\infty (q^{24}; q^{24})_\infty \\ &= (q^6, q^{12}, q^{18}, q^{24}; q^{24})_\infty\end{aligned} \tag{1.28}$$

$$\begin{aligned}(q^6; q^{12})_\infty &= (q^6; q^{60})_\infty (q^{18}; q^{60})_\infty (q^{30}; q^{60})_\infty (q^{42}; q^{60})_\infty (q^{54}; q^{60})_\infty \\ &= (q^6, q^{18}, q^{30}, q^{42}, q^{54}; q^{60})_\infty\end{aligned} \tag{1.29}$$

$$\begin{aligned}(q^8;q^8)_\infty &= (q^8;q^{24})_\infty(q^{16};q^{24})_\infty(q^{24};q^{24})_\infty \\ &= (q^8,q^{16},q^{24};q^{24})_\infty\end{aligned} \tag{1.30}$$

$$\begin{aligned}(q^8;q^8)_\infty &= (q^8;q^{48})_\infty(q^{16};q^{48})_\infty(q^{24};q^{48})_\infty(q^{32};q^{48})_\infty(q^{40};q^{48})_\infty(q^{48};q^{48})_\infty \\ &= (q^8,q^{16},q^{24},q^{32},q^{40},q^{48};q^{48})_\infty\end{aligned} \tag{1.31}$$

$$\begin{aligned}(q^8;q^{12})_\infty &= (q^8;q^{60})_\infty(q^{20};q^{60})_\infty(q^{32};q^{60})_\infty(q^{44};q^{60})_\infty(q^{56};q^{60})_\infty \\ &= (q^8,q^{20},q^{32},q^{44},q^{56};q^{60})_\infty\end{aligned} \tag{1.32}$$

$$\begin{aligned}(q^8;q^{16})_\infty &= (q^8;q^{48})_\infty(q^{24};q^{48})_\infty(q^{40};q^{48})_\infty \\ &= (q^8,q^{24},q^{40};q^{48})_\infty\end{aligned} \tag{1.33}$$

$$\begin{aligned}(q^{10};q^{20})_\infty &= (q^{10};q^{60})_\infty(q^{30};q^{60})_\infty(q^{50};q^{60})_\infty \\ &= (q^{10},q^{30},q^{50};q^{60})_\infty\end{aligned} \tag{1.34}$$

$$\begin{aligned}(q^{12};q^{12})_\infty &= (q^{12};q^{24})_\infty(q^{24};q^{24})_\infty \\ &= (q^{12},q^{24};q^{24})_\infty\end{aligned} \tag{1.35}$$

$$\begin{aligned}(q^{12};q^{12})_\infty &= (q^{12};q^{60})_\infty(q^{24};q^{60})_\infty(q^{36};q^{60})_\infty(q^{48};q^{60})_\infty(q^{60};q^{60})_\infty \\ &= (q^{12},q^{24},q^{36},q^{48},q^{60};q^{60})_\infty\end{aligned} \tag{1.36}$$

$$\begin{aligned}(q^{16};q^{16})_\infty &= (q^{16};q^{48})_\infty(q^{32};q^{48})_\infty(q^{48};q^{48})_\infty \\ &= (q^{16},q^{32},q^{48};q^{48})_\infty\end{aligned} \tag{1.37}$$

$$\begin{aligned}(q^{20};q^{20})_\infty &= (q^{20};q^{60})_\infty(q^{40};q^{60})_\infty(q^{60};q^{60})_\infty \\ &= (q^{20},q^{40},q^{60};q^{60})_\infty\end{aligned} \tag{1.38}$$

The outline of this paper is as follows. In sections 2, we have recorded some well known results on continued fraction identities and recent results on character formulas for mock theta functions of Ramanujan given by Folsom [2], those are useful to the rest of the paper. In section 3, we obtain fourteen new results.

2 Preliminaries and Statement of Results

The famous Rogers-Ramanujan continued fraction identity [3, (1.6)], is

$$\frac{(q;q^5)_\infty(q^4;q^5)_\infty}{(q^2;q^5)_\infty(q^3;q^5)_\infty} = \cfrac{1}{1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{1+\cfrac{q^4}{1+\vdots}}}}} \tag{2.1}$$

In 1983 Denis [5], has introduced following continued fraction identity

$$(q^2;q^2)_\infty(-q;q)_\infty = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} = \cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}} \tag{2.2}$$

another well known continued fraction identity due to Ramanujan [4, (4.21)], is

$$\frac{(-q^3;q^4)_\infty}{(-q;q^4)_\infty} = \cfrac{1}{1+\cfrac{q}{1+\cfrac{q^3+q^2}{1+\cfrac{q^5}{1+\cfrac{q^7+q^4}{1+\cfrac{q^9}{1+\cfrac{q^{11}+q^6}{1+\vdots}}}}}}} \tag{2.3}$$

One of the most celebrated continued fractional identities associated with Ramanujan's academic career, given by Rogers-Ramanujan, is

$$C(q) = \frac{(q^2;q^5)_\infty(q^3;q^5)_\infty}{(q;q^5)_\infty(q^4;q^5)_\infty} = 1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{1+\cfrac{q^4}{1+\cfrac{q^5}{1+\vdots}}}}} \tag{2.4}$$

Folsom in Table 1 [2; p. 450], recorded character formulas for order 3 mock θ - functions. We are giving below only those functions, which having terms of q-product identities, as

$$\begin{aligned} f(-q) = &-4q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0})+ \\ &+4q\widehat{\beta}_{12,1}(\tau) + \frac{(q^2;q^2)_\infty^7}{(q;q)_\infty^3(q^4;q^4)_\infty^3} \end{aligned} \tag{2.5}$$

$$\begin{aligned} \phi(q) = &-2q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0})+ \\ &+2q\widehat{\beta}_{12,1}(\tau) + \frac{(q^2;q^2)_\infty^7}{(q;q)_\infty^3(q^4;q^4)_\infty^3} \end{aligned} \tag{2.6}$$

$$\begin{aligned} \chi(-q) = &-q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0})+ \\ &+q\widehat{\beta}_{12,1}(\tau) + \frac{(q^4;q^4)_\infty^3(q^6;q^6)_\infty^3}{(q^2;q^2)_\infty^2(q^3;q^3)_\infty(q^{12};q^{12})_\infty^2} \end{aligned} \tag{2.7}$$

$$\begin{aligned} \upsilon(q) = &-q.\widehat{\Theta}_{12}^{-1}(tr_{L(\bigwedge_{(-2)};\,13)}q^{L_0} + tr_{L(\bigwedge_{(2)};\,13)}q^{L_0})+ \\ &+q\widehat{\beta}_{12,-2}(\tau) + \frac{(q^4;q^4)_\infty^3}{(q^2;q^2)_\infty^2} \end{aligned} \tag{2.8}$$

$$\rho(q) = -\frac{1}{2}.\widehat{\Theta}_6^{-1}(tr_{L(\bigwedge_{(-1)};\,7)}q^{L_0} + tr_{L(\bigwedge_{(1)};\,7)}q^{L_0})+$$

$$+\frac{1}{2}\widehat{\beta}_{6,-1}(\tau)+\frac{3}{2}\frac{(q^6;q^6)^4_\infty}{(q^2;q^2)_\infty(q^3;q^3)^2_\infty} \tag{2.9}$$

$$\sigma(-q)=q^2.\widehat{\Theta}^{-1}_{36}(q^{\frac{3}{2}}.tr_{L(\bigwedge_{(3)};\,37)}q^{L_0}+q^{-\frac{3}{2}}.tr_{L(\bigwedge_{(15)};\,37)}q^{L_0})-$$

$$-q^2\widehat{\beta}_{36,3}(\tau)+\frac{(q^2;q^2)^3_\infty(q^{12};q^{12})^3_\infty}{(q;q)_\infty(q^4;q^4)^2_\infty(q^6;q^6)^2_\infty} \tag{2.10}$$

Folsom in Table 4 [2; p. 452], recorded character formulas for order 2 mock θ - functions. We are giving below only those functions, which having terms of q-product identities, as

$$A(q^2)=q.\widehat{\Theta}^{-1}_8 tr_{L(\bigwedge_{(2)};\,9)}q^{L_0}-q\widehat{\eta}_{8,2}(\tau)-q(-q^2;q^2)_\infty(-q^4;q^4)^2_\infty(q^8;q^8)_\infty \tag{2.11}$$

$$\mu(q^4)=-2q.\widehat{\Theta}^{-1}_4 tr_{L(\bigwedge_{(0)};\,5)}q^{L_0}+2q\widehat{\eta}_{4,0}(\tau)+$$

$$+\frac{(q^2;q^2)_\infty(q^4;q^4)^3_\infty(q^8;q^8)_\infty}{(q;q)^2_\infty(q^{16};q^{16})^2_\infty}\times 12 \tag{2.12}$$

Folsom in Table 5 [2; p. 452], recorded character formulas for order 6 mock θ - functions. We are giving below both the functions, which having terms of q-product identities, as

$$\phi(q^4)=-2q.\widehat{\Theta}^{-1}_{12} tr_{L(\bigwedge_{(4)};\,13)}q^{L_0}+2q\widehat{\eta}_{12,4}(\tau)+$$

$$+\frac{(q^2;q^2)^3_\infty(q^3;q^3)^2_\infty(q^{12};q^{12})^3_\infty}{(q;q)^2_\infty(q^6;q^6)^3_\infty(q^8;q^8)_\infty(q^{24};q^{24})_\infty} \tag{2.13}$$

$$\psi(q^4)=-q^3.\widehat{\Theta}^{-1}_{12} tr_{L(\bigwedge_{(0)};\,13)}q^{L_0}+q^3\widehat{\eta}_{12,0}(\tau)+$$

$$+q^3\frac{(q^2;q^2)^2_\infty(q^4;q^4)_\infty(q^{24};q^{24})^2_\infty}{(q;q)_\infty(q^3;q^3)_\infty(q^8;q^8)^2_\infty} \tag{2.14}$$

Folsom in Table 7 [2; p. 453], recorded character formulas for order 10 mock θ - functions. We are giving below all the four functions, which having terms of q-product identities, as

$$\phi(q)=2q.\widehat{\Theta}^{-1}_{10} tr_{L(\bigwedge_{(1)};\,11)}q^{L_0}-2q\widehat{\eta}_{10,1}(\tau)+\frac{(q^{10};q^{10})^2_\infty j(-q^2;q^5)}{(q^5;q^5)_\infty j(q^2;q^{10})} \tag{2.15}$$

$$\psi(q)=2q.\widehat{\Theta}^{-1}_{10} tr_{L(\bigwedge_{(3)};\,11)}q^{L_0}-2q\widehat{\eta}_{10,3}(\tau)-q\frac{(q^{10};q^{10})^2_\infty j(-q;q^5)}{(q^5;q^5)_\infty j(q^4;q^{10})} \tag{2.16}$$

$$X(-q^2)=-2q.\widehat{\Theta}^{-1}_{40} tr_{L(\bigwedge_{(18)};\,41)}q^{L_0}+2q.\widehat{\Theta}^{-1}_{40} tr_{L(\bigwedge_{(2)};\,41)}q^{L_0}+$$

$$+2q\widehat{\eta}_{40,18}(\tau)-2q\widehat{\eta}_{40,2}(\tau)+$$

$$+\frac{(q^4;q^4)^2_\infty(j(-q^2,q^{20})j(q^{12},q^{40})+2q(q^{40};q^{40})^3_\infty)}{(q^2;q^2)_\infty(q^{20};q^{20})_\infty(q^{40};q^{40})_\infty j(q^8,q^{40})} \tag{2.17}$$

Here, we have written above equations with minor corrections, which was occurred in original published paper may be due to printing error.

$$\chi(-q^2)=-2q^3.\widehat{\Theta}^{-1}_{40} tr_{L(\bigwedge_{(14)};\,41)}q^{L_0}-2q^5.\widehat{\Theta}^{-1}_{40} tr_{L(\bigwedge_{(6)};\,41)}q^{L_0}+$$

$$+2q^3\widehat{\eta}_{40,14}(\tau)+2q^5\widehat{\eta}_{40,6}(\tau)+$$

$$+q^2\frac{(q^4;q^4)^2_\infty(2q(q^{40};q^{40})^3_\infty-j(-q^6,q^{20})^2j(q^4,q^{40}))}{(q^2;q^2)_\infty(q^{20};q^{20})_\infty(q^{40};q^{40})_\infty j(q^{16},q^{40})} \tag{2.18}$$

3 Main Results

In this section, we obtain representations for character formulas in terms of continued fraction identities or in more precise language, we can say an applications of continued fraction identities to character formulas given by Folsom [2]. We obtain fourteen new results parallel to character formulas (2.5) to (2.18), which are recorded in [2, pp. 450, and 452-453], using q-product identities given in (2.12) to (1.38).

$$
\begin{aligned}
f(-q) = &- 4q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0}) + 4q\widehat{\beta}_{12,1}(\tau)+\\
&+\frac{(q^2;q^2)_\infty(q^2,q^6,q^8,q^{10},q^{12},q^{14},q^{18};q^{20})_\infty(q^2,q^6,q^8,q^{10},q^{14};q^{16})^2_\infty}{(q^4,q^{16};q^{20})_\infty(q^{16};q^{16})^2_\infty}\times\\
&\times\left[\cfrac{1}{1-\cfrac{q^8}{1+\cfrac{q^8(1-q^8)}{1-\cfrac{q^{24}}{1+\cfrac{q^{16}(1-q^{16})}{1-\cfrac{q^{40}}{1+\cfrac{q^{24}(1-q^{24})}{1+\vdots}}}}}}}\right]^2\times\left[\cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}}\right]^3\times\\
&\times\cfrac{1}{1+\cfrac{q^4}{1+\cfrac{q^8}{1+\cfrac{q^{12}}{1+\cfrac{q^{16}}{1+\cfrac{q^{20}}{1+\cfrac{q^{24}}{1+\vdots}}}}}}}
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
\phi(q) = &- 2q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0}) + 2q\widehat{\beta}_{12,1}(\tau)+\\
&+\frac{(q^2;q^2)_\infty(q^2,q^6,q^8,q^{10},q^{12},q^{14},q^{18};q^{20})_\infty(q^2,q^6,q^8,q^{10},q^{14};q^{16})^2_\infty}{(q^4,q^{16};q^{20})_\infty(q^{16};q^{16})^2_\infty}\times\\
&\times\left[\cfrac{1}{1-\cfrac{q^8}{1+\cfrac{q^8(1-q^8)}{1-\cfrac{q^{24}}{1+\cfrac{q^{16}(1-q^{16})}{1-\cfrac{q^{40}}{1+\cfrac{q^{24}(1-q^{24})}{1+\vdots}}}}}}}\right]^2\times\left[\cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}}\right]^3\times\\
&\times\cfrac{1}{1+\cfrac{q^4}{1+\cfrac{q^8}{1+\cfrac{q^{12}}{1+\cfrac{q^{16}}{1+\cfrac{q^{20}}{1+\cfrac{q^{24}}{1+\vdots}}}}}}}
\end{aligned}
\tag{3.2}
$$

$$\chi(-q) = -\,q.\widehat{\Theta}_{12}^{-1}\left(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0}\right)+$$

$$+\,q\widehat{\beta}_{12,1}(\tau) + \frac{(q^4;q^4)_\infty(q^6;q^6)_\infty}{(q^{12};q^{12})_\infty^2(q^2,q^6,q^{14},q^{18},q^{20};q^{20})_\infty^2}\times$$

$$\times\cfrac{1}{1-\cfrac{q^3}{1+\cfrac{q^3(1-q^3)}{1-\cfrac{q^9}{1+\cfrac{q^6(1-q^6)}{1-\cfrac{q^{15}}{1+\cfrac{q^9(1-q^9)}{1+\vdots}}}}}}}\times\left[\cfrac{1}{1-\cfrac{q^{10}}{1+\cfrac{q^{10}(1-q^{10})}{1-\cfrac{q^{30}}{1+\cfrac{q^{20}(1-q^{20})}{1-\cfrac{q^{50}}{1+\cfrac{q^{30}(1-q^{30})}{1+\vdots}}}}}}}\right]^2\times$$

$$\times\left[\cfrac{1}{1+\cfrac{q^4}{1+\cfrac{q^8}{1+\cfrac{q^{12}}{1+\cfrac{q^{16}}{1+\cfrac{q^{20}}{1+\cfrac{q^{24}}{1+\vdots}}}}}}}\right]^2\times\left[1+\cfrac{q^4}{1+\cfrac{q^8}{1+\cfrac{q^{12}}{1+\cfrac{q^{16}}{1+\cfrac{q^{20}}{1+\cfrac{q^{24}}{1+\cfrac{q^{28}}{1+\vdots}}}}}}}\right]^2 \tag{3.3}$$

$$v(q) = -\,q.\widehat{\Theta}_{12}^{-1}\left(tr_{L(\bigwedge_{(-2)};\,13)}q^{L_0} + tr_{L(\bigwedge_{(2)};\,13)}q^{L_0}\right) + \;q\widehat{\beta}_{12,-2}(\tau)+$$

$$+\frac{(q^4;q^4)_\infty}{(q^2,q^6,q^{14},q^{18},q^{20};q^{20})_\infty^2}\times\left[\cfrac{1}{1-\cfrac{q^{10}}{1+\cfrac{q^{10}(1-q^{10})}{1-\cfrac{q^{30}}{1+\cfrac{q^{20}(1-q^{20})}{1-\cfrac{q^{50}}{1+\cfrac{q^{30}(1-q^{30})}{1+\vdots}}}}}}}\right]^2\times$$

$$\times\left[\cfrac{1}{1+\cfrac{q^4}{1+\cfrac{q^8}{1+\cfrac{q^{12}}{1+\cfrac{q^{16}}{1+\cfrac{q^{20}}{1+\cfrac{q^{24}}{1+\vdots}}}}}}}\right]^2\times\left[1+\cfrac{q^4}{1+\cfrac{q^8}{1+\cfrac{q^{12}}{1+\cfrac{q^{16}}{1+\cfrac{q^{20}}{1+\cfrac{q^{24}}{1+\cfrac{q^{28}}{1+\vdots}}}}}}}\right]^2 \tag{3.4}$$

$$\rho(q) = -\,\frac{1}{2}.\widehat{\Theta}_6^{-1}\left(tr_{L(\bigwedge_{(-1)};\,7)}q^{L_0} + tr_{L(\bigwedge_{(1)};\,7)}q^{L_0}\right) + \frac{1}{2}\widehat{\beta}_{6,-1}(\tau) + \frac{3}{2}\frac{1}{(q^2;q^2)_\infty}\times$$

$$\times \left[\cfrac{1}{1-\cfrac{q^3}{1+\cfrac{q^3(1-q^3)}{1-\cfrac{q^9}{1+\cfrac{q^6(1-q^6)}{1-\cfrac{q^{15}}{1+\cfrac{q^9(1-q^9)}{1+\vdots}}}}}}}\right]^2 \tag{3.5}$$

$$\sigma(-q) = q^2.\widehat{\Theta}_{36}^{-1}(q^{\frac{3}{2}}.tr_{L(\bigwedge_{(3)};\,37)}q^{L_0} + q^{-\frac{3}{2}}.tr_{L(\bigwedge_{(15)};\,37)}q^{L_0}) - \ q^2\widehat{\beta}_{36,3}(\tau) +$$
$$+\frac{(q^2,q^{10};q^{12})_\infty^2(q^6;q^{12})_\infty}{(q;q^2)_\infty} \times \cfrac{1}{1-\cfrac{q^6}{1+\cfrac{q^6(1-q^6)}{1-\cfrac{q^{18}}{1+\cfrac{q^{12}(1-q^{12})}{1-\cfrac{q^{30}}{1+\cfrac{q^{18}(1-q^{18})}{1+\vdots}}}}}}} \tag{3.6}$$

$$A(q^2) = q.\widehat{\Theta}_8^{-1}tr_{L(\bigwedge_{(2)};\,9)}q^{L_0} - q\widehat{\eta}_{8,2}(\tau) - q(-q^2;q^2)_\infty(-q^4;q^4)_\infty \times$$
$$\times \cfrac{1}{1-\cfrac{q^4}{1+\cfrac{q^4(1-q^4)}{1-\cfrac{q^{12}}{1+\cfrac{q^8(1-q^8)}{1-\cfrac{q^{20}}{1+\cfrac{q^{12}(1-q^{12})}{1+\vdots}}}}}}} \tag{3.7}$$

$$\mu(q^4) = -\,2q.\widehat{\Theta}_4^{-1}tr_{L(\bigwedge_{(0)};\,5)}q^{L_0} + 2q\widehat{\eta}_{4,0}(\tau) + \frac{(q^8;q^8)_\infty}{(q;q)_\infty(q^2;q^4)_\infty} \times$$
$$\times \cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}} \times 12 \tag{3.8}$$

$$\phi(q^4) = -\,2q.\widehat{\Theta}_{12}^{-1}tr_{L(\bigwedge_{(4)};\,13)}q^{L_0} + 2q\widehat{\eta}_{12,4}(\tau) + \frac{(q^2,q^4,q^6;q^8)_\infty(q^{12};q^{24})_\infty(q^3;q^6)_\infty^2}{(q;q)_\infty} \times$$
$$\times \cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}} \times \cfrac{1}{1-\cfrac{q^6}{1+\cfrac{q^6(1-q^6)}{1-\cfrac{q^{18}}{1+\cfrac{q^{12}(1-q^{12})}{1-\cfrac{q^{30}}{1+\cfrac{q^{18}(1-q^{18})}{1+\vdots}}}}}}} \tag{3.9}$$

$$\psi(q^4) = -q^3.\widehat{\Theta}_{12}^{-1} tr_{L(\bigwedge_{(0)};\,13)} q^{L_0} + q^3 \widehat{\eta}_{12,0}(\tau) + \frac{(q^4,q^{12},q^{20},q^{24};q^{24})_\infty}{(q^8,q^{16};q^{24})_\infty} \times$$

$$\times \frac{q^3}{(q^3;q^3)_\infty} \times \cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}} \tag{3.10}$$

$$\phi(q) = 2q.\widehat{\Theta}_{10}^{-1} tr_{L(\bigwedge_{(1)};\,11)} q^{L_0} - 2q\widehat{\eta}_{10,1}(\tau) + \frac{j(-q^2;q^5)}{j(q^2;q^{10})} \times$$

$$\times \cfrac{1}{1-\cfrac{q^5}{1+\cfrac{q^5(1-q^5)}{1-\cfrac{q^{15}}{1+\cfrac{q^{10}(1-q^{10})}{1-\cfrac{q^{25}}{1+\cfrac{q^{15}(1-q^{15})}{1+\vdots}}}}}}} \tag{3.11}$$

$$\psi(q) = 2q.\widehat{\Theta}_{10}^{-1} tr_{L(\bigwedge_{(3)};\,11)} q^{L_0} - 2q\widehat{\eta}_{10,3}(\tau) - q\frac{j(-q;q^5)}{j(q^4;q^{10})} \times$$

$$\times \cfrac{1}{1-\cfrac{q^5}{1+\cfrac{q^5(1-q^5)}{1-\cfrac{q^{15}}{1+\cfrac{q^{10}(1-q^{10})}{1-\cfrac{q^{25}}{1+\cfrac{q^{15}(1-q^{15})}{1+\vdots}}}}}}} \tag{3.12}$$

$$X(-q^2) = -2q.\widehat{\Theta}_{40}^{-1} tr_{L(\bigwedge_{(18)};\,41)} q^{L_0} + 2q.\widehat{\Theta}_{40}^{-1} tr_{L(\bigwedge_{(2)};\,41)} q^{L_0} + 2q\widehat{\eta}_{40,18}(\tau) -$$

$$-2q\widehat{\eta}_{40,2}(\tau) + \frac{(j(-q^2,q^{20})j(q^{12},q^{40}) + 2q(q^{40};q^{40})_\infty^3)}{(q^{20};q^{20})_\infty (q^{40};q^{40})_\infty j(q^8,q^{40})} \times$$

$$\times \cfrac{1}{1-\cfrac{q^2}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^6}{1+\cfrac{q^4(1-q^4)}{1-\cfrac{q^{10}}{1+\cfrac{q^6(1-q^6)}{1+\vdots}}}}}}} \tag{3.13}$$

$$\chi(-q^2) = -2q^3.\widehat{\Theta}_{40}^{-1} tr_{L(\bigwedge_{(14)};\,41)} q^{L_0} - 2q^5.\widehat{\Theta}_{40}^{-1} tr_{L(\bigwedge_{(6)};\,41)} q^{L_0} + 2q^3\widehat{\eta}_{40,14}(\tau) +$$

$$+2q^5\widehat{\eta}_{40,6}(\tau) + q^2 \frac{(2q(q^{40};q^{40})_\infty^3 - j(-q^6,q^{20})^2 j(q^4,q^{40}))}{(q^{20};q^{20})_\infty (q^{40};q^{40})_\infty j(q^{16},q^{40})} \times$$

$$\times \cfrac{1}{1-\cfrac{q^2}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^6}{1+\cfrac{q^4(1-q^4)}{1-\cfrac{q^{10}}{1+\cfrac{q^6(1-q^6)}{1+\vdots}}}}}}} \tag{3.14}$$

Proof of (3.1): To prove this result, we start with (2.5), as given below

$$f(-q) = -4q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0})\, 4q\widehat{\beta}_{12,1}(\tau) + \\ + \frac{(q^2;q^2)_\infty^7}{(q;q)_\infty^3 (q^4;q^4)_\infty^3}$$

now, we make suitable rearrangement only in the part related to q-product identities, and keep rest part is unchanged in the above expression, as

$$f(-q) = -4q.\widehat{\Theta}_{12}^{-1}(q^{\frac{1}{2}}.tr_{L(\bigwedge_{(1)};\,13)}q^{L_0} + q^{-\frac{1}{2}}.tr_{L(\bigwedge_{(5)};\,13)}q^{L_0})\, 4q\widehat{\beta}_{12,1}(\tau) + \\ + \frac{(q^2;q^2)_\infty^2}{(q^4;q^4)_\infty^2} \times \frac{(q^2;q^2)_\infty^3}{(q;q)_\infty^3} \times \frac{(q^2;q^2)_\infty}{(q^4;q^4)_\infty} \times (q^2;q^2)_\infty$$

further, using q-product identities given in (1.19),(1.20),(1.23) and (1.24), and further applying continued fractional identities given in (2.1) and (2.2), after little algebra we get (3.1).

Proof of (3.2): Proof of (3.2) is similar as (3.1), as q-products identities are same in both expression.

Proofs of (3.3) to (3.14): Proofs for (3.3) to (3.14), can be obtain on similar lines as (3.1) and (3.2) by using suitable q-product identities listed in section 2 of this paper. We are leaving it for the readers.

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