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Topological cordial labeling of some graphs

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Abstract

B.D. Acharya [3] introduced the notion of set - valuation as set analogue of number valuation as introduced by A. Rosa [5]. Let G be a graph and X, a non-empty set. Define an injective function $f : V(G) \to 2^X$ such that $\{f(V(G))\}$ is a topology on X. If the induced function f^* on $E(G)$ is defined by

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1 then f is a topological cordial labeling and a graph which admits such a labeling is called topological cordial graph. This definition is defined and introduced by us [8]. In this paper we proved Durer graph, Herchel graph and some constructed graphs are topolological cordial graph.

Keywords

Durer graph, Herchel graph, Wheel graph, Octahedral graph and topolological cordial graph.

AMS Subject Classification

05C78, 68Q45.

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1. Introduction

The graphs treated in this paper are simple. For standard terminology and notations we follow F. Harary [4]. Given a graph $G = (V, E)$, we can relate it to different topological structures. The relation between topology and graph theory is undergone many investigations. In 1983 Acharya [3] established another link between graph theory and point - set topology. He defined a set - indexer as follows: Let $G = (V, E)$ be a graph, X any non $-$ empty set and 2^x denote the set of all subsets of X. A set - indexer of G is an injective set valued function $f : V(G) \to 2^X$ such that the induced function $f^* : E(G) \to 2^X - \{\phi\}$ defined by $f^*(v_1v_2) = f(v_1) \Delta f(v_2)$ for every $v_1v_2 \in E(G)$ is also injective, where Δ denotes the symmetric difference of sets. A graph $G = (V, E)$ is said to be a bitopological graph if there exist a set indexer $f : V(G) \to 2^X$ such that $f(V)$ and $f^*(E) \cup \{\phi\}$ are both topologies on the corresponding ground set. Let G be a graph and X, a non-empty set. Define an injective function $f : V(G) \to 2^X$ such that $\{f(V(G))\}$ is a topology on X. If the induced function f^* on $E(G)$ is defined by

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and} \\ & \text{singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1 then f is a topological cordial labeling and a graph which admits such a labeling is called topological cordial graph. This definition is defined and introduced by us [8]. In this paper we proved Durer graph, Herchel graph and some constructed graphs are topolological cordial graph.

Definition 1.1 ([6]). *The Durer graph is an undirected graph with 12 vertices and 18 edges.*

Definition 1.2 ([6]). *The Herschel graph H is a bipartite graph with 11 vertices and 18 edges, the smallest non Hamiltonian polyhedr*

Definition 1.3 ([6]). *The octahedral graph is the 6 -node and 12 -edge platonic graph having the connectivity of the octahedron.*

Definition 1.4 ([8]). *Let G be a graph and X, a non-empty set. Define an injective function $f : V(G) \to 2^X$ such that $\{f(V(G))\}$ is a topology on X. If the induced function f^* on $E(G)$ is defined by*

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and} \\ & \text{singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \le 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1 then f is a topological cordial labeling and a graph which admits such a labeling is called topological cordial graph.

2. Topological cordial labeling of named graphs

Theorem 2.1. *Durer graph is topological cordial graph.*

Proof. Let G be a Durer graph with 12 vertices and 18 edges.

Let $V(G) = \{u_i/1 \le i \le 6\} \cup \{v_i/1 \le i \le 6\}$ and $E(G) = \{v_iv_{i+1}/1 \le i \le 6$ where $v_7 = v_1\} \cup \{u_iu_{i+2}/1 \le i \le 6\}$ where $u_7 = u_1$ and $u_8 = u_2\} \cup \{v_iu_i/1 \le i \le 6\}$. Let $X = \{1,2,\ldots,11\}$. Now, define $f : V(G) \to 2^X$ by $f(u_1) = \phi, f(u_2) = \{1\}, f(u_3) = \{2\}, f(u_4) = \{1,2\} f(u_5) = \{1,2,3\}, f(u_6) = \{1,3\}, f(v_i) = \{1,2,\ldots,i+3\}, 1 \le i \le 5, f(v_6) = X$.

Then the vertex labels are distinct and $\{f(V(G))\ \}$is a topology on X. The induced function f^* on $E(G)$ is defined as follows:

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and} \\ & \text{singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(G)$. Now, $f^*(v_iv_{i+1}) = 1, 1 \le i \le 6$ where $v_7 = v_1, f^*(u_iv_i) = 0, 1 \le i \le 3, f^*(u_iv_i) = 1, 4 \le i \le 6, f^*(u_iu_{i+2}) = 0, 1 \le i \le 6$ where $u_7 = u_1$ and $u_8 = u_2$.

Therefore, $|e_f(0) - e_f(1)| \le 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1.Hence f is a topological cordial labeling. Thus G is topological cordial graph. □

Example 2.2. *Durer graph is topological cordial graph.*

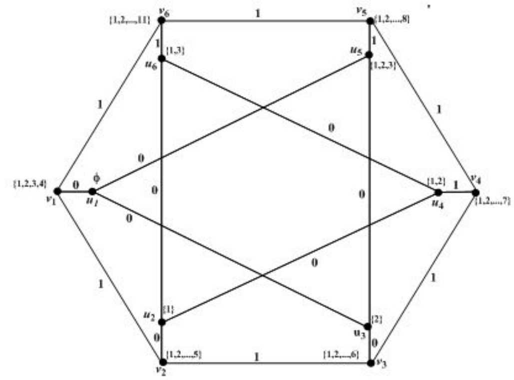

Figure 1

Theorem 2.3. *The Herschel graph H is topological cordial graph.*

Proof. Let H be a Herschel graph.
Let $V(H) = \{v_i/1 \le i \le 11\}$ and $E(H) = \{v_1v_i/1 \le i \le 8$ where i is even $\} \cup \{v_iv_{11}/i = 3,5,7\} \cup \{v_iv_{10}/i = 3,7\} \cup \{v_2v_9\} \cup \{v_iv_{i+1}/2 \le i \le 9\}$. Then $|V(H)| = 11$ and $|E(H)| = 18$. Let $X = \{1,2,3,\ldots,10\}$. Define $f : V(G) \to 2^X$ by $f(v_1) = \phi, f(v_2) = \{1,2,3\}, f(v_3) = \{1,2,3,4\}, f(v_4) = \{1,2,3,4,5\}, f(v_5) = \{1\}, f(v_6) = \{1,2,\ldots,6\}.f(v_7) = \{1,2\}, f(v_8) = \{1,2,\ldots,7\}, f(v_9) = \{1,2,\ldots,8\}, f(v_{10}) = \{1,2,\ldots,10\}, f(v_{11}) = \{2\}$. Therefore the vertex labels are distinct and $\{f(V(H))\ \}$ is a topology on X. The induced function f^* on $E(H)$ is defined as follows:

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u) \cap f(v) \text{ is not an empty set and} \\ & \text{singleton set} \\ 0 & \text{otherwise} \end{cases}$$

for every $uv \in E(H)$. Then, $|e_f(0) - e_f(1)| = 9 - 9 = 0 \le 1$ where $e_f(0)$ = number of edges labeled with 0 and $e_f(1)$ = number of edges labeled with 1. Hence f is a topological cordial labeling. Thus H is topological cordial graph. □

Example 2.4. *Herschel graph H is topological cordial graph.*

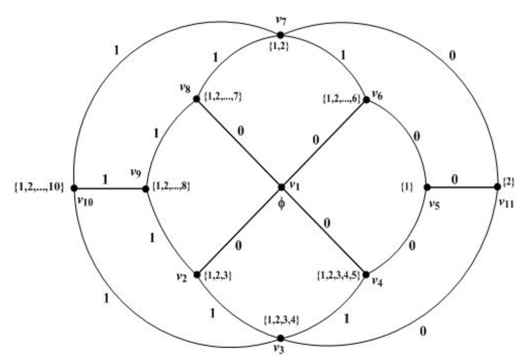

Figure 2

3. Topological cordial labeling of constructed graphs

Theorem 3.1. *A wheel graph W_n together with a new vertex attached to the centre vertex is a topological cordial graph.*

Proof. Let G be a wheel graph W_n together with a new vertex attached to the centre vertex.Let $v_1, v_2, \ldots, v_n$ be the vertices of W_n other than the centre vertex v_0 and w be the new vertex attached with the centre vertex. Let $V(G)=\{v_i/0\leq i\leq n\}\cup\{w\}$ and $E(G)=\{v_0v_i/1\leq i\leq n\}\cup\{v_iv_{i+1}/1\leq i\leq n$ where $v_{n+1}=v_1\}\cup\{v_0w\}$.

Then the graph G has $n+2$ vertices and $2n+1$ edges. Let $X=\{1,2,\ldots,n+1\}$.

Define $f:V(G)\to 2^X$ by f $(v_0)=\phi_2 f(v_i)=\{1,2,\ldots,i+1\}, 1\leq i\leq n, f(w)=\{1\}$. Then the vertex labels are distinct and $\{f(V(G))\}$ is a topology on X.The induced function f^* on $E(G)$ is defined as follows:

$$f^*(uv)=\begin{cases}1 & \text{if } f(u)\cap f(v) \text{ is not an empty set and} \\ & \text{singleton set} \\ 0 & \text{otherwise}\end{cases}$$

for every $uv\in E(G)$.$f^*(v_0v_i)=0, 1\leq i\leq n, f^*(v_iv_{i+1})=1$, $1\leq i\leq n$ where $v_{n+1}=v_1, f^*(v_0w)=0$.

Therefore, $\left|e_f(0)-e_f(1)\right|\leq 1$ where $e_f(0)=$ number of edges labeled with 0 ande $_f(1)=$ number of edges labeled with 1. Hence f is a topological cordial labeling. Thus G is topological cordial graph. □

Example 3.2. *A wheel graph W_{10} together with a new vertex attached to the centre vertex is a topological cordial graph.*

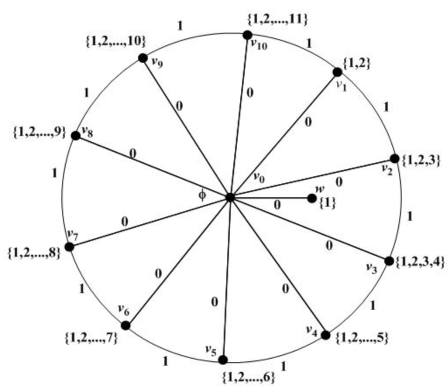

Figure 3

Theorem 3.3. *An Octahedral graph in which each rim vertex is attached to a new vertex is a topological cordial graph.*

Proof. Let G be a Octahedral graph in which each rim vertex is attached to a new vertex. Thus it has 9 vertices and 15 edges.Let $V(G)=\{v_i/1\leq i\leq 6\}\cup\{u_i/1\leq i\leq 3\}$ an $E(G)=\{v_iv_{i+1}/1\leq i\leq 6$ where $v_7=v_4\}\cup\{u_iv_{i+3}/1\leq i\leq 3\}\cup\{v_1v_i/i=4,6\}\cup\{v_2v_i/i=5,6\}\cup\{v_3v_i/i=1,5\}$. Let $X=\{1,2,\ldots,8\}$ Define $f:V(G)\to 2^X$ by $f(v_1)=\phi, f(v_{i+1})=\{1,2,\ldots,i\}, 1\leq i\leq 5, f(u_i)=\{1,2,\ldots,i+5\}, 1\leq i\leq 3$. Then the vertex labels are distinct and $\{f(V(G))\}$ is a topology on X. The induced function f^* on $E(G)$ is defined as follows:

$$f^*(uv)=\begin{cases}1 & \text{if } f(u)\cap f(v) \text{ is not an empty set and} \\ & \text{singleton set} \\ 0 & \text{otherwise}\end{cases}$$

for every $uv\in E(G)$.Then, $\left|e_f(0)-e_f(1)\right|\leq 1$ where $e_f(0)=$ number of edges labeled with 0 and $e_f(1)=$ number of edges labeled with 1 .Hence f is a topological cordial labeling. Thus G is topological cordial graph. □

Example 3.4. *Octahedral graph in which each rim vertex is attached to a new vertex is a topological cordial graph.*

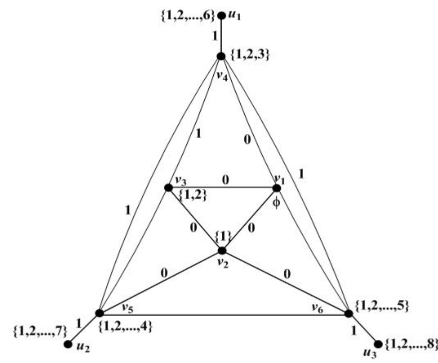

Figure 4

4. Conclusion

In this paper deals with topological cordial graphs. The aim of this paper is to make some progress to a better understanding of topological cordial labeling.

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