\(\mu_{ij}\)-semi open sets in bigeneralized topological space

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Abstract

In this paper, we introduce \(\mu_{ij}\)-semi open sets and study its properties in bigeneralized topological spaces.

Keywords

Generalized topology, bigeneralized topology, \(\mu\)-open set, \(\mu\)-closed set, \(\mu_{ij}\)-semi open set, \(\mu_{ij}\)-semi closed set.

AMS Subject Classification

18B30.

1. Introduction

A. Csaszar[2] introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In [7] Sivagami studied the properties of \(\gamma\)-Semi open sets. C. Boonpok [1] introduced the concept of bigeneralized topological spaces and studied \((m,n)\)-closed sets and \((m,n)\)-open sets in bigeneralized topological spaces.

In this paper we introduce the notions of \(\mu_{ij}\)-semi open sets in bigeneralized topological spaces and study some of their properties. Also we investigate some of their characterizations.

2. Preliminaries

We recall some basic definitions and notations. Let 

\(X\) be a set and denotes \(\exp X\) the power set of \(X\). A subset \(\mu\) of \(\exp X\) is said to be generalized topology (briefly GT) on \(X\) if \(\phi \in \mu\) and an arbitrary union of elements of \(\mu\) belongs to \(\mu\)[3]. Let \(\mu\) be a GT on \(X\), the elements of \(\mu\) are called \(\mu\)-open sets and the complements of \(\mu\)-open sets are called \(\mu\)-closed sets. Let \(M_\mu\) denote the union of all elements of \(\mu\). We say \(\mu\) is strong [4] if \(M_\mu = X\). If \(A \subseteq X\) then \(i_\mu(A)\) denotes the union of all \(\mu\)-open sets contained in \(A\) and \(c_\mu(A)\) is the intersections of all \(\mu\)-closed sets containing \(A\)[5]. According to [7], for \(A \subseteq X\) and \(x \in X\) we have \(x \in c_\mu(A)\) if and only if \(x \in M \subseteq \mu\) implies \(M \cap A \neq \phi\). Let \((X, \mu)\) be a generalized topological space. A subset \(M \subseteq X\) is said to be \(\mu\)-open if and only if \(M \subseteq c_\mu(i_\mu(M))\), \(\mu\)-preopen if and only if \(M \subseteq i_\mu(c_\mu(i_\mu(M)))\), \(\mu\)-\(\alpha\)-open if and only if \(M \subseteq i_\mu(c_\mu(i_\mu(M)))\), finally \(\mu\)-\(\beta\)-open if and only if \(M \subseteq c_\mu(i_\mu(c_\mu(M)))\)[5].

Theorem 2.1. [2] Let \((X, \mu)\) be a generalized topological space. Then

1. \(c_\mu(A) = X - i_\mu(X - A)\);
2. \(i_\mu(A) = X - c_\mu(X - A)\)

Proposition 2.2. [6] Let \((X, \mu)\) be a generalized topological space. For subsets \(A\) and \(B\) of \(X\), the following properties holds:

1. \(c_\mu(X - A) = X - i_\mu(A)\) and \(i_\mu(X - A) = X - c_\mu(A)\).
2. If \((X - A) \in \mu\), then \(c_\mu(A) = A\) and if \(A \in \mu\), then \(i_\mu(A) = A\);
3. If \(A \subseteq B\), then \(c_\mu(A) \subseteq c_\mu(B)\) and \(i_\mu(A) \subseteq i_\mu(B)\);
4. \(A \subseteq c_\mu(A)\) and \(i_\mu(A) \subseteq A\);
Theorem 3.4. \[
A \subset \bigcup_{\mu} \text{denoted by} \ i_{\mu} (A) \text{and only if} \ i_{\mu}(A) = i_{\mu}(A).
\]

Definition 2.3. \[\text{Let} \ X \text{be a nonempty set and} \ \mu_1, \mu_2 \text{be}
\]

generalized topologies on \(X\). \(A\) triple \((X, \mu_1, \mu_2)\) is said to be a

bigereneralized topological space.

Let \((X, \mu_1, \mu_2)\) is said to be a bigeneralized topological space and \(A\) a subset of \(X\). The closure of \(A\) and the interior of \(A\) with respect to \(\mu_m\) are denoted by \(c_{\mu_m}(A)\) and \(i_{\mu_m}(A)\), respectively, for \(m = 1, 2\).

Definition 2.4. \[\text{A subset} \ A \text{of a bigeneralized topological}
\]

space \((X, \mu_1, \mu_2)\) is called \((m, n)\)-closed if and only if \(c_{\mu_m}(i_{\mu_n}(A)) = A\), where \(m, n = 1, 2\) and \(m \neq n\). The complement of \((m, n)\)-closed set is \((m, n)\)-open.

Proposition 2.5. \[\text{Let} \ (X, \mu_1, \mu_2) \text{be a bigeneralized topological}
\]

space and \(A\) a subset of \(X\). Then \(A\) is \((m, n)\)-closed if and only if \(A\) is both \(\mu\)-closed in \((X, \mu_m)\) and \((X, \mu_n)\).

Proposition 2.6. \[\text{Let} \ (X, \mu_1, \mu_2) \text{be a bigeneralized topological}
\]

space. Then \(A\) is \((m, n)\)-open if and only if \(i_{\mu_m}(i_{\mu_n}(A)) = A\).

3. \(\mu_{ij}\)-semi open sets

Definition 3.1. \[\text{Let} \ (X, \mu_1, \mu_2) \text{be a bigeneralized topological}
\]

space. Let \(A\) a subset of \(X\). Then \(A\) is said to be \(\mu_{ij}\)-semi open if \(A \subset C_{\mu}(i_{\mu_j}(A))\) where \(i, j = 1, 2\) and \(i \neq j\). The complement of \(\mu_{ij}\)-semi open set is called \(\mu_{ij}\)-semi closed set. The collection of all \(\mu_{ij}\)-semi open sets is denoted by \(\sigma_{ij}(\mu)\).

Example 3.2. \[\text{Let} \ X = \{a, b, c\}. \text{Consider two generalized}
\]

topologies \(\mu_1 = \{\phi, \{a\}, \{b\}, \{a, b\}\}\) and \(\mu_2 = \{\phi, \{a\}, \{c\}, \{a, c\}\}\) on \(X\). Then \(\{a\}\) is \(\mu_{12}\)-semi open but \(\{a, b, c\}\) is not \(\mu_{12}\)-semi open.

Definition 3.3. \[\text{Let} \ (X, \mu_1, \mu_2) \text{be a bigeneralized topological}
\]

space. Let \(A\) a subset of \(X\). Then the union of all \(\mu_{ij}\)-semi open sets contained in \(A\) is called the \(\mu_{ij}\)-semi interior of \(A\) and is denoted by \(i_{\sigma_{ij}}(A)\). The intersection of all \(\mu_{ij}\)-semi closed sets containing \(A\) is called the \(\mu_{ij}\)-semi closure of \(A\) and is denoted by \(C_{\sigma_{ij}}(A)\).

The following Theorem 3.6 gives some of the properties of \(\mu_{ij}\)-semi interior operator \(i_{\sigma_{ij}}\).

Theorem 3.6. \[\text{Let} \ (X, \mu_1, \mu_2) \text{be a bigeneralized topological}
\]

space. Let \(A\) a subset of \(X\). Then

(a) \(i_{\sigma_{ij}}(A)\) is the largest \(\mu_{ij}\)-semi open set contained in \(A\).

(b) \(A\) is \(\mu_{ij}\)-semi open if and only if \(A = i_{\sigma_{ij}}(A)\).

(c) \(x \in i_{\sigma_{ij}}(A)\) if and only if there exist \(\mu_{ij}\)-semi open \(G\) containing \(x\) such that \(G \subset A\).

(d) \(i_{\sigma_{ij}}(A) = \Gamma_{012}\).

Theorem 3.7. \[\text{Let} \ (X, \mu_1, \mu_2) \text{be a bigeneralized topological}
\]

space. Let \(A\) a subset of \(X\). Then the following hold.

(a) \((i_{\sigma_{ij}})^* = C_{\sigma_{ij}}\).

(b) \((C_{\sigma_{ij}})^* = i_{\sigma_{ij}}\).

(c) \(i_{\sigma_{ij}}(X - A) = X - C_{\sigma_{ij}}(A)\) for every subset \(A\) of \(X\).

(d) \(C_{\sigma_{ij}}(X - A) = X - i_{\sigma_{ij}}(A)\) for every subset \(A\) of \(X\).

Proof.

(a) \(A\) be a subset of \(X\). Then \((i_{\sigma_{ij}})^*(A) = X - i_{\sigma_{ij}}(X - A)\). Since \(i_{\sigma_{ij}}(X - A)\) is the largest \(\mu_{ij}\)-semi open set contained in \(X - A, X - i_{\sigma_{ij}}(X - A)\) is the smallest \(\mu_{ij}\)-semi closed set containing \(A\) and so \(X - i_{\sigma_{ij}}(X - A) = C_{\sigma_{ij}}(A)\). Hence \((i_{\sigma_{ij}})^* = C_{\sigma_{ij}}\).

(b) \((C_{\sigma_{ij}})^* = ((i_{\sigma_{ij}})^*)^* = i_{\sigma_{ij}}\). This proves (b).
(c). Let $A$ be a subset of $X$. Then $(i_{\sigma_j})^* = X - i_{\sigma_j}(X - A)$ and by (b), $C_{\mu_j}(A) = X - i_{\sigma_j}(X - A)$ which implies that $i_{\sigma_j}(X - A) = X - C_{\sigma_j}(A)$ for every subset $A$ of $X$.

(d) The proof is similar to the proof of (c). \end{proof}

**Theorem 3.8.** Let $(X, \mu_1, \mu_2)$ be a bigeneralized topological space. Let $A$ be a subset of $X$. Then the following are equivalent.

(a) $A$ is $\mu_{ij}$-semi open

(b) $A \subset C_{\mu_j}(i_{\mu_j}(A))$

(c) $C_{\mu_j}(A) = C_{\mu_j}(i_{\mu_j}(A))$

**Proof.** (a) $\Rightarrow$ (b).

Suppose $A$ is $\mu_{ij}$-semi open. Then there exist a $\mu_{ij}$-open set $G$ such that $G \subset A \subset C_{\mu_j}(i_{\mu_j}(G))$. Since $G$ is $\mu_{ij}$-open, $G = i_{\mu_j}(G)$ and so $A \subset C_{\mu_j}(i_{\mu_j}(G))$. Since $G \subset A$, $i_{\mu_j}(G) \subset i_{\mu_j}(A)$. Then $C_{\mu_j}(i_{\mu_j}(G)) \subset C_{\mu_j}(i_{\mu_j}(A))$. Therefore, $A \subset C_{\mu_j}(i_{\mu_j}(A))$.

(b) $\Rightarrow$ (c).

Suppose $A \subset C_{\mu_j}(i_{\mu_j}(A))$. Then $C_{\mu_j}(A) \subset C_{\mu_j}(i_{\mu_j}(A))$. Since $i_{\mu_j}(A) \subset A$, $C_{\mu_j}(i_{\mu_j}(A)) \subset C_{\mu_j}(A)$. Thus $C_{\mu_j}(A) = C_{\mu_j}(i_{\mu_j}(A))$.

(c) $\Rightarrow$ (a).

Suppose $C_{\mu_j}(i_{\mu_j}(A)) = C_{\mu_j}(A)$. Since $i_{\mu_j}(A)$ is a $\mu_{ij}$-open set such that $i_{\mu_j}(A) \subset A \subset C_{\mu_j}(i_{\mu_j}(A))$. Therefore there exist $\mu_{ij}$-open set $i_{\mu_j}(A)$ such that $i_{\mu_j}(A) \subset A \subset C_{\mu_j}(i_{\mu_j}(A))$. Hence $A$ is $\mu_{ij}$-semi open. \end{proof}

**Theorem 3.9.** Let $(X, \mu_1, \mu_2)$ be a bigeneralized topological space. Let $A$ be a subset of $X$. Then the following hold.

(a) $A$ is $\mu_{ij}$-semi open if and only if $A$ is $\mu_{ij}i_{\mu_j}i_{\mu_j}$-open if and only if $A = i_{C_{\mu_j}(A)}(A)$

(b) $i_{\sigma_j} = i_{C_{\mu_j}(A)}$ and $C_{\sigma_j} = C_{i_{\mu_j}(A)}$

(c) $C_{\sigma_j}(A) = A \cap C_{\mu_j}(i_{\mu_j}(A))$

**Proof.** (a) follows from Theorem 3.8 (a) and (b).

(b) If $x \in i_{\sigma_j}(A)$ then there exist a $\mu_{ij}$-semi open set $B$ such that $x \in B \subset A$. Then by (a) $B$ is $\mu_{ij}i_{\mu_j}$-open and $x \in i_{C_{\mu_j}(B)}(A)$. Hence $i_{\sigma}(A) \subset i_{C_{\mu_j}(B)}(A)$. Similarly, we can prove that $i_{C_{\mu_j}(B)}(A) \subset i_{\sigma_j}(A)$.

(c) $i_{\sigma_j}(A) = A \cap C_{\mu_j}(i_{\mu_j}(A))$. Also by (b) $i_{\sigma_j}(A) = i_{\mu_j}(i_{\mu_j}(A))$.

Therefore, $i_{\sigma_j}(A) = A \cap C_{\mu_j}(i_{\mu_j}(A))$

(d) By (c), $i_{\mu_j}(i_{\mu_j}(A)) = A \cap C_{\mu_j}(i_{\mu_j}(A))$. Then $(i_{\mu_j}(i_{\mu_j}(A)))^* = (A \cap C_{\mu_j}(i_{\mu_j}(A)))^*$ implies $C_{\mu_j}(i_{\mu_j}(A)) = A \cap i_{\mu_j}(C_{\mu_j}(i_{\mu_j}(A)))$. Also by (b), $C_{\mu_j}(A) = A \cup i_{\mu_j}(C_{\mu_j}(A))$. \end{proof}

**Theorem 3.10.** Let $(X, \mu_1, \mu_2)$ be a bigeneralized topological space. Let $A \subset B \subset C_{\mu_j}(A)$ and $A$ is $\mu_{ij}$-semi open. Then $B$ is $\mu_{ij}$-semi open.

**Proof.** Suppose $A$ is $\mu_{ij}$-semi open. Then by Theorem 3.8(c), $C_{\mu_j}(A) = C_{\mu_j}(i_{\mu_j}(A))$. Since $B \subset C_{\mu_j}(A)$, $B \subset C_{\mu_j}(i_{\mu_j}(B))$ and so by Theorem 3.8 (b), $B$ is $\mu_{ij}$-semi open. \end{proof}

**Theorem 3.11.** Let $(X, \mu_1, \mu_2)$ be a bigeneralized topological space. Let $A$ be a subset of $X$. Then the following are equivalent.

(a) $A$ is $\mu_{ij}$-semi closed

(b) $i_{\mu_j}(C_{\mu_j}(A)) \subset A$

(c) $i_{\mu_j}(C_{\mu_j}(A)) = i_{\mu_j}(A)$

(d) There exist a $\mu_j$ closed set $F$ (resp. $\mu_i$-closed sets) such that $i_{\mu_j}(F) \subset A \subset F$

**Proof.** (a) $\Rightarrow$ (b).

If $A$ is $\mu_{ij}$-semi closed, then $X - A$ is $\mu_{ij}$-semi open and so by Theorem 3.8(b) $X - A \subset C_{\mu_j}(i_{\mu_j}(X - A))$. Also $C_{\mu_j}(X - A) = X - i_{\mu_j}(C_{\mu_j}(A))$ and so $i_{\mu_j}(C_{\mu_j}(A)) \subset A$.

(b) $\Rightarrow$ (c).

If $i_{\mu_j}(C_{\mu_j}(A)) \subset A$, then $i_{\mu_j}(C_{\mu_j}(A)) \subset i_{\mu_j}(A)$ and so $i_{\mu_j}(C_{\mu_j}(A)) = i_{\mu_j}(A)$.

(c) $\Rightarrow$ (d).

Since $X - F = C_{\mu_j}(A)$, then $F$ is a $\mu_j$-closed set (resp. $\mu_i$). Now $i_{\mu_j}(F) = i_{\mu_j}(C_{\mu_j}(A)) = i_{\mu_j}(A) \subset A \subset F$ which proves (d).

(d) $\Rightarrow$ (a).

Suppose there exist a $\mu_j$-closed set $F$ (resp. $\mu_i$) such that $i_{\mu_j}(F) \subset A \subset F$. Then $X - F \subset X - A \subset X - i_{\mu_j}(F) = C_{\mu_j}(X - F)$. Since $X - F$ is $\mu_{ij}$-semi open, $X - A$ is $\mu_{ij}$-semi open and $A$ is $\mu_{ij}$-semi closed. \end{proof}

**Definition 3.12.** We say that $A$ is $C_{\mu_j}$ dense if $C_{\mu_j}(A) = X$.

**Theorem 3.13.** Let $(X, \mu_1, \mu_2)$ be a bigeneralized topological space. Let $A$ be a subset of $X$. Then the following are equivalent.

(a) $C_{\mu_j}(A) = X$

(b) $C_{\sigma_j}(A) = X$

(c) If $B$ is any $\mu_{ij}$-semi closed subset of $X$ such that $A \subset B$, then $B = X$

(d) Every non empty $\mu_{ij}$-semi open set has a non empty intersection with $A$

(e) $i_{\sigma_j}(X - A) = \phi$.

**Proof.** (a) $\Rightarrow$ (b).

Suppose $x \notin C_{\sigma_j}(A)$. Then by Theorem 3.5(c), there exist a $\mu_{ij}$-semi open set $G$ containing $X$ such that $G \cap A = \phi$. Since $G$ is a nonempty $\mu_{ij}$-semi open set, there exist $\mu_i$ open set $H$ such that $H \subset G \subset C_{\mu_i}(H)$ and $H \cap A = \phi$ which implies $C_{\mu_i}(A) \neq X$, a contradiction. Hence $C_{\sigma_j}(A) = X$

(b) $\Rightarrow$ (c).

If $B$ is any $\mu_{ij}$-semi closed subset of $X$ such that $A \subset B$, then
$X = C_{\sigma_i}(A) \subset C_{\sigma_j}(B) = B$ and so $X = B$.

(c) $\Rightarrow$ (d).

If $G$ is any nonempty $\mu_{ij}$-semi open set such that $G \cap A = \phi$. Then $A \subset X - G$ and $X - G$ is $\mu_{ij}$-semi closed which implies $X - G = X$ and so $G = \phi$, a contradiction. Therefore $G \cap A \neq \phi$.

(d) $\Rightarrow$ (e).

If $i_{\sigma_i}(X - A) \neq \phi$. Then $i_{\sigma_i}(X - A)$ is a nonempty $\mu_{ij}$-semi open set such that $i_{\sigma_i}(X - A) \cap A \neq \phi$, a contradiction. Therefore $i_{\sigma_i}(X - A) = \phi$.

(e) $\Rightarrow$ (a).

Suppose $i_{\sigma_i}(X - A) = \phi$. Then $X - i_{\sigma_i}(X - A) = X$ and so $C_{\sigma_i}(A) = X$. Also $C_{\sigma_i}(A) \subset C_{\mu_i}(A)$ (resp. $\mu_j$) for any subset $A$ of $X$ and hence $C_{\sigma_i}(A) = X$ implies $C_{\mu_i}(A) = X$. □

### References


