



Superiority and inferiority ranking method with hesitant Pythagorean fuzzy set for solving MCDM problems

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Abstract

In this research article, we aims to present a novel MCDM method based on the hesitant Pythagorean fuzzy set (HPFS). With the greater extension of solution space of HPFS we can able to solve MCDM problems effectively. In the proposed method we extend the superiority and inferiority ranking (SIR) method with HPFS. The superiority and inferiority ranking method helps to decision makers to obtain effective results with intelligible computation. The efficiency of the proposed method expressed by illustrative numerical example.

Keywords

Superiority, Inferiority, Hesitant, Pythagorean, MCDM.

AMS Subject Classification

03E72, 68U35, 90B50, 91B06 .

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Contents

| | | |
|---|---|----|
| 1 | Introduction | 11 |
| 2 | Preliminaries | 12 |
| 3 | Hesitant Pythagorean Fuzzy- Superiority and Inferiority Ranking Method..... | 12 |
| 4 | Example | 13 |
| 5 | Conclusion | 15 |
| | References | 15 |

1. Introduction

The superiority and inferiority ranking (SIR) method introduced by Xu in 2001 [15]. The SIR method ranking the alternatives based on the two ranking list. In this method alternatives are ranked by superiority ranking list and inferiority ranking list. The main advantage to utilize SIR method is which combines the properties of other MCDM methods namely TOPSIS, SAW and PROMETHEE. Tam et al., [12] utilized SIR method for selecting concrete pump in 2004. Tom and Tong [11] engaged SIR method for project developments regarding locating the large-scale harbor in 2008. Liu [7] proposed SIR method with intuitionistic fuzzy set to solve supply chain management problem in 2010. Ma et al., [8]

extended SIR method with hesitant fuzzy set and they also introduced interval- valued hesitant fuzzy SIR method in 2014. Peng and Yang [9] introduced Pythagorean fuzzy SIR method and also they proved some results on Pythagorean fuzzy set in 2015. Rouhani [10] employed fuzzy SIR method for software selection in IT field. Chen [2] introduced Pythagorean fuzzy PROMETHEE method with superiority and inferiority Pythagorean fuzzy numbers for solving real life problem regarding the bridge construction methods in 2018. Tavana et al., [13] solved third-party reverse logistics problem with IFG - SIR method in 2018. Zhao et al., utilized SIR method with HFL prioritized value in 2019. Jian et al., [19] solved investment selection problem with IVIF - SIR method in 2019.

The decision makers can able to solve the MCDM problem with wide solution space in Pythagorean fuzzy environment [5]. With the Pythagorean fuzzy set experts expressing their opinions in the different element of fuzzy set [3], [6]. But in the group decision making situation we are going with the possibility of giving experts opinion in the single element [18]. The HFS is more convenient to express the experts knowledge in single element[4]. So when we are working with HPFS, we possess both advantages of hesitant fuzzy set and Pythagorean fuzzy set [16].

From the motivation of the above discussion we intro-

duced SIR method with HPFS. In HPF - SIR method, experts able to solve MCDM problems three advantages like alternatives are analyzed exactly in the superiority and inferiority flow and potential to handle experts preference values in single element and obtaining solution with intelligible solution space of Pythagorean fuzzy set.

The remaining sections of this research article is structured as follows. In section 2, we reviews some basic definitions related to HPF - SIR method. In section 3, we proposed SIR method with HPFS. In section 4, we solved a numerical example problem to express the efficiency of the proposed method. Finally we concluded this article with section 5.

| Nomenclature: | |
|---------------|--|
| IFS | Intuitionistic Fuzzy Set |
| IVIF | Interval-Valued Intuitionistic Fuzzy |
| HFS | Hesitant Fuzzy Set |
| IFG | Intuitionistic Fuzzy-Grey |
| HFL | Hesitant Fuzzy Linguistic |
| PHS | Pythagorean Fuzzy Set |
| HPFS | Hesitant Pythagorean Fuzzy Set |
| MCDM | Multi- Criteria Decision Making |
| SIR | Superiority and Inferiority Ranking |
| PROMETHEE | Preference Ranking Organization METHod for Enrichment of Evaluation |
| TOPSIS | Technique for Order Preference by Similarity to Ideal Solution |
| SAW | Simple Additive Weighting |

2. Preliminaries

In this section, we reviews some definitions and basic concepts related to HPF - SIR method.

Definition 2.1. [1] An intuitionistic fuzzy set J on the universal set G is mathematically represents as follows:

$$J = \{ \langle \xi, \omega_J(\xi), \epsilon_J(\xi) \rangle / \xi \in G \},$$

where the functions $\omega_J(\xi) : G \rightarrow [0, 1]$ and $\epsilon_J(\xi) : G \rightarrow [0, 1]$ represent the membership degree and non- membership degree, respectively, of the element $\xi \in G$, which satisfying the condition, $0 \leq \omega_J(\xi) + \epsilon_J(\xi) \leq 1$.

Definition 2.2. [14] Let G be a fixed set. Then the hesitant fuzzy set on G is a subset of $[0, 1]$. The hesitant fuzzy set has the following form:

$$A = (\langle \xi, \alpha(\xi) \rangle / \xi \in G),$$

where $\alpha(\xi)$ is a hesitant fuzzy element, which has a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $\xi \in G$ in the set A .

Definition 2.3. [17] A Pythagorean fuzzy set B on G is represented using the two functions $\omega(\xi)$, $\epsilon(\xi)$. Mathematically, it is represented by following expression:

$$B = \{ \langle \xi, \omega_B(\xi), \epsilon_B(\xi) \rangle, \forall \xi \in G \},$$

where $\omega_B(\xi) : G \rightarrow [0, 1]$ and $\epsilon_B(\xi) : G \rightarrow [0, 1]$ represent the possible membership degrees and non-membership degrees, respectively, of the elements $\xi \in G$ to the set B . The Pythagorean fuzzy set satisfies the following conditions:

$$0 \leq (\omega_B(\xi))^2 + (\epsilon_B(\xi))^2 \leq 1$$

Definition 2.4. [?] A hesitant Pythagorean fuzzy set C on G is represented by the two functions $c_1(\xi)$, $c_2(\xi)$. Mathematically, it can be represented by the following expression:

$$C = \{ \langle \xi, c_1(\xi), c_2(\xi) \rangle, \forall \xi \in G \},$$

where $c_1(\xi) : G \rightarrow [0, 1]$ and $c_2(\xi) : G \rightarrow [0, 1]$ both are represents the possible membership degrees and non-membership degrees of the elements $\xi \in G$ to the set C . The hesitant Pythagorean fuzzy set satisfies the following conditions:

$$\omega \geq 0, \quad \epsilon \leq 1, \quad 0 \leq (\omega^+)^2 + (\epsilon^+)^2 \leq 1, \\ \forall \omega \in c_1(\xi), \epsilon \in c_2(\xi)$$

where ω^+ and ϵ^+ are represented as follows:

$$\omega^+ \in c_1^+(\xi) = \bigcup_{\omega \in c_1(\xi)} \max\{\omega\} \quad \forall \xi \in G, \\ \epsilon^+ \in c_2^+(\xi) = \bigcup_{\epsilon \in c_2(\xi)} \max\{\epsilon\} \quad \forall \xi \in G.$$

Definition 2.5. [6] Let $\xi = \{ \omega, \epsilon \}$, $\xi_1 = \{ \omega_1, \epsilon_1 \}$ and $\xi_2 = \{ \omega_2, \epsilon_2 \}$ be any three hesitant Pythagorean fuzzy sets. The basic operations on hesitant Pythagorean fuzzy sets are as follows:

Union: $\xi_1 \cup \xi_2 = (\max\{\omega_1, \omega_2\}, \min\{\epsilon_1, \epsilon_2\})$

Intersection: $\xi_1 \cap \xi_2 = (\min\{\omega_1, \omega_2\}, \max\{\epsilon_1, \epsilon_2\})$

Addition: $\xi_1 \oplus \xi_2 = (\sqrt{\omega_1^2 + \omega_2^2 - \omega_1^2 \omega_2^2}, \epsilon_1^2 \epsilon_2^2)$

Multiplication: $\xi_1 \otimes \xi_2 = (\omega_1^2 \omega_2^2, \sqrt{\epsilon_1^2 + \epsilon_2^2 - \epsilon_1^2 \epsilon_2^2})$

Scalar multiplication: $\lambda \xi = (\sqrt{1 - (1 - \omega^2)^\lambda}, \epsilon^\lambda), \lambda > 0$

Power multiplication: $\xi^\lambda = (\omega^\lambda, \sqrt{1 - (1 - \epsilon^2)^\lambda}), \lambda > 0$

Complement: $\xi^c = (\epsilon, \omega)$

3. Hesitant Pythagorean Fuzzy-Superiority and Inferiority Ranking Method

In this section, we proposed hesitant Pythagorean fuzzy superiority and inferiority ranking method. Algorithm of the extended method is given as follows. Let us consider the set of alternatives $M_i = \{M_1, M_2, \dots, M_i\}$ ($i = 1, 2, \dots, m$) and set of criteria $N_j = \{N_1, N_2, \dots, N_j\}$ ($j = 1, 2, \dots, n$). The decision makers are $D_k = \{d_1, d_2, \dots, d_l\}$ ($l = 1, 2, \dots, k$) and weight



vector of criteria is $W_j = \{w_1, w_2, \dots, w_j\}$ ($j = 1, 2, \dots, n$). The decision matrix elements having the form:

$$\xi_{ij} = \left\{ \omega_{ij}^l, \varepsilon_{ij}^l / \omega_{ij}, \varepsilon_{ij} \in \xi_{ij} \right\} \quad (3.1)$$

The hesitant Pythagorean fuzzy decision matrix Ξ_{ij} is defined as follows.

$$\Xi = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} & \dots & \xi_{1n} \\ \xi_{21} & \xi_{22} & \xi_{23} & \dots & \xi_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \xi_{m1} & \xi_{m2} & \xi_{m3} & \dots & \xi_{mn} \end{bmatrix}$$

Step 1: The decision makers weight vector is $\zeta_l = \{\zeta_1, \zeta_2, \dots, \zeta_l\}$ ($l = 1, 2, \dots, k$) which is also represented by hesitant Pythagorean fuzzy number.

Calculate the individual preference of the experts by using the relative closeness coefficient value. The weight of each experts are determined by using the following equation:

$$\tilde{\zeta}_l = \frac{d(\zeta_l, \zeta^-)}{d(\zeta_l, \zeta^-) + d(\zeta_l, \zeta^+)} \quad (3.2)$$

where $\zeta^+ = (\max\{\omega_l\}, \min\{\varepsilon_l\})$ and $\zeta^- = (\min\{\omega_l\}, \max\{\varepsilon_l\})$. Then the normalized weight of each experts are obtained as follows:

$$\zeta_l = \frac{\tilde{\zeta}_l}{\sum_{l=1}^k \tilde{\zeta}_l} \quad (3.3)$$

Step 2: Aggregate experts opinion from the hesitant Pythagorean fuzzy numbers to single element by using the following aggregation operator.

$$\tilde{\xi}_{ij} = \oplus (\xi_{ij}^1, \xi_{ij}^2, \dots, \xi_{ij}^k) = \left(\sum_{l=1}^k \zeta_l \omega_{ij}^l, \sum_{l=1}^k \zeta_l \varepsilon_{ij}^l \right) \quad (3.4)$$

where $(\omega_{ij}^l, \varepsilon_{ij}^l) \in \xi_{ij}^l$. Aggregate weight of each criteria from the hesitant Pythagorean fuzzy numbers to single element as follows:

$$W_j = \oplus (w_1, w_2, \dots, w_j) = \left(\sum_{l=1}^k \zeta_l \omega_j^l, \sum_{l=1}^k \zeta_l \varepsilon_j^l \right) \quad (3.5)$$

where $(\omega_j^l, \varepsilon_j^l) \in w_j^l$.

Step 3: Determine the performance function by using following equation:

$$\Gamma_{ij} = \frac{d(\tilde{\xi}_{ij}, \tilde{\xi}_{ij}^-)}{d(\tilde{\xi}_{ij}, \tilde{\xi}_{ij}^-) + d(\tilde{\xi}_{ij}, \tilde{\xi}_{ij}^+)} \quad (3.6)$$

where $\tilde{\xi}_{ij}^+ = \{N_j, \max\{\omega_{ij}\}, \min\{\varepsilon_{ij}\} / (\omega_{ij}, \varepsilon_{ij}) \in \tilde{\xi}_{ij}\}$ and $\tilde{\xi}_{ij}^- = \{N_j, \min\{\omega_{ij}\}, \max\{\varepsilon_{ij}\} / (\omega_{ij}, \varepsilon_{ij}) \in \tilde{\xi}_{ij}\}$.

Step 4: Calculate the preference intensity $\lambda_j(M_i, M_m)$ of the

alternative M_i over alternative M_m with respect to the criteria j .

$$\lambda_j(M_i, M_m) = \delta_j(\Gamma_{ij} - \Gamma_{mj}) = \delta_j(\beta) \quad (3.7)$$

Step 5: Determine superiority and inferiority matrices Ψ_{ij} and Λ_{ij} respectively as follows:

Superiority matrix:

$$\Psi_{ij} = \sum_{\forall m, m \neq i} \lambda_j(M_i, M_m) = \sum_{\forall m, m \neq i} \delta_j(\Gamma_{ij} - \Gamma_{mj}) \quad (3.8)$$

Inferiority matrix:

$$\Lambda_{ij} = \sum_{\forall m, m \neq i} \lambda_j(M_m, M_i) = \sum_{\forall m, m \neq i} \delta_j(\Gamma_{mj} - \Gamma_{ij}) \quad (3.9)$$

Step 6: Determine superiority flow and inferiority flow as follows:

Superiority flow:

$$\Phi_i^> = \left(\sum_{j=1}^n \left(\Psi_{ij} * \sum_{l=1}^k \zeta_l \omega_j^l \right), \sum_{j=1}^n \left(\Psi_{ij} * \sum_{l=1}^k \zeta_l \varepsilon_j^l \right) \right) \quad (3.10)$$

Inferiority flow:

$$\Phi_i^< = \left(\sum_{j=1}^n \left(\Lambda_{ij} * \sum_{l=1}^k \zeta_l \omega_j^l \right), \sum_{j=1}^n \left(\Lambda_{ij} * \sum_{l=1}^k \zeta_l \varepsilon_j^l \right) \right) \quad (3.11)$$

Step 7: Rank the alternatives by using the following superiority and inferiority ranking rules.

SR-Rule:

1. If $\Phi_i^> > \Phi_m^>$ and $\Phi_i^< < \Phi_m^<$, then $M_i > M_m$;
2. If $\Phi_i^> > \Phi_m^>$ and $\Phi_i^< = \Phi_m^<$, then $M_i > M_m$;
3. If $\Phi_i^> = \Phi_m^>$ and $\Phi_i^< < \Phi_m^<$, then $M_i > M_m$;

IR-Rule:

1. If $\Phi_i^> < \Phi_m^>$ and $\Phi_i^< > \Phi_m^<$, then $M_i < M_m$;
2. If $\Phi_i^> < \Phi_m^>$ and $\Phi_i^< = \Phi_m^<$, then $M_i < M_m$;
3. If $\Phi_i^> = \Phi_m^>$ and $\Phi_i^< > \Phi_m^<$, then $M_i < M_m$;

4. Example

Considers MCDM problem for selecting the internet stock marketing companies for investment [9]. The organization has the following three experts as decision committee members: d_1 —market maker, d_2 —dealer and d_3 —finder to find potential investment value. The alternatives are: M_1 —SINA, M_2 —BIDU, M_3 —NETS and M_4 —BABA. The criteria for evaluate the companies are: N_1 — stock market trend, N_2 — policy direction and N_3 — annual performance.



| Alternatives | Criteria | | |
|--------------|--|--|--|
| | N_1 | N_2 | N_3 |
| M_1 | $\{\{0.6, 0.5, 0.7\}, \{0.4, 0.3, 0.2\}\}$ | $\{\{0.7, 0.8, 0.7\}, \{0.2, 0.2, 0.3\}\}$ | $\{\{0.3, 0.5, 0.4\}, \{0.7, 0.5, 0.6\}\}$ |
| M_2 | $\{\{0.3, 0.4, 0.2\}, \{0.6, 0.5, 0.7\}\}$ | $\{\{0.4, 0.2, 0.5\}, \{0.5, 0.6, 0.3\}\}$ | $\{\{0.7, 0.8, 0.6\}, \{0.3, 0.1, 0.2\}\}$ |
| M_3 | $\{\{0.6, 0.4, 0.4\}, \{0.4, 0.4, 0.6\}\}$ | $\{\{0.5, 0.4, 0.3\}, \{0.5, 0.6, 0.6\}\}$ | $\{\{0.4, 0.5, 0.3\}, \{0.6, 0.4, 0.5\}\}$ |
| M_4 | $\{\{0.3, 0.3, 0.2\}, \{0.7, 0.6, 0.8\}\}$ | $\{\{0.3, 0.2, 0.3\}, \{0.7, 0.8, 0.7\}\}$ | $\{\{0.2, 0.3, 0.2\}, \{0.7, 0.7, 0.8\}\}$ |

Table 1. Evaluation of alternatives under the criteria

The hesitant Pythagorean fuzzy decision matrix is given in table 1.

The preference weight vector of experts:

$$\zeta_1 = (0.8, 0.3), \quad \zeta_2 = (0.9, 0.4), \quad \zeta_3 = (0.7, 0.2)$$

The weight vector of criteria:

$$w_1 = \{(0.8, 0.9, 0.6), (0.3, 0.1, 0.2)\}$$

$$w_2 = \{(0.7, 0.8, 0.9), (0.2, 0.3, 0.2)\}$$

$$w_3 = \{(0.6, 0.8, 0.8), (0.4, 0.1, 0.2)\}$$

Calculate the individual preference of the experts by using equation 3.2:

$$\tilde{\zeta}_1 = 0.5, \quad \tilde{\zeta}_2 = 0.7272, \quad \tilde{\zeta}_3 = 0.2727$$

Then the normalized weight of each experts are obtained by using equation 3.3:

$$\varsigma_1 = 0.3333, \quad \varsigma_2 = 0.4848, \quad \varsigma_3 = 0.1818$$

The aggregation of decision makers opinion by using equation 3.4 given in table 2.

| Alternatives | Criteria | | |
|--------------|----------|--------|--------|
| | N_1 | N_2 | N_3 |
| M_1 | 0.6667 | 0.6555 | 0.6778 |
| M_2 | 0.5778 | 0.6394 | 0.7161 |
| M_3 | 0.7030 | 0.6040 | 0.6172 |
| M_4 | 0.5768 | 0.5949 | 0.7111 |

Table 2. Aggregation of HPFS

Aggregate weight of each criteria by using equation 3.5:

$$W_1 = 0.3322, \quad W_2 = 0.3444, \quad W_3 = 0.3171$$

Determine the performance function by using equation 3.6:

$$\Gamma_{ij} = \begin{bmatrix} 0.748 & 1 & 0.6475 \\ 0.0094 & 0.7492 & 1 \\ 1 & 0.1604 & 0 \\ 0 & 0 & 0.9562 \end{bmatrix}$$

Calculate the preference intensity $\lambda_j(M_i, M_m)$ of the alternative M_i over alternative M_m with respect to the criteria j by

using equation 3.7. Here

$$\delta_j(\beta) = \begin{cases} 0.01 & \text{if } \beta > 0 \\ 0.00 & \text{if } \beta \leq 0 \end{cases}$$

The superiority matrix:

$$\Psi_{ij} = \begin{bmatrix} 0.02 & 0.03 & 0.01 \\ 0.03 & 0.01 & 0.00 \\ 0.00 & 0.00 & 0.02 \\ 0.01 & 0.02 & 0.03 \end{bmatrix}$$

The inferiority matrix:

$$\Lambda_{ij} = \begin{bmatrix} 0.01 & 0.00 & 0.02 \\ 0.00 & 0.02 & 0.03 \\ 0.03 & 0.03 & 0.01 \\ 0.02 & 0.01 & 0.00 \end{bmatrix}$$

Rank the alternatives by using the superiority and inferiority ranking rules.

| Alternatives | $\Phi_i^>$ | $\Phi_i^<$ | Ranking |
|--------------|------------|------------|---------|
| M_1 | 0.0201 | 0.0096 | 1 |
| M_2 | 0.0134 | 0.0164 | 3 |
| M_3 | 0.0063 | 0.0234 | 4 |
| M_4 | 0.0197 | 0.0101 | 2 |

Table 3. Ranking list of alternatives

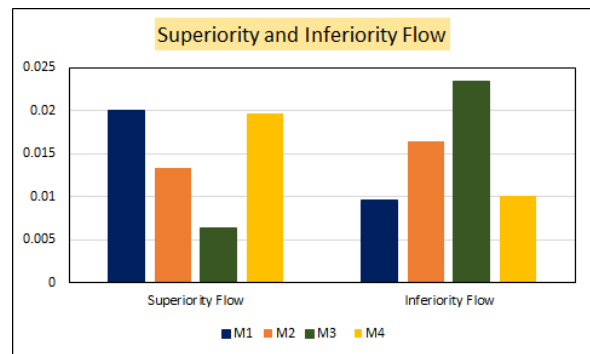


Figure 1. Superiority and Inferiority Flow

The figure 1 clearly explains the alternatives ranking order. In which the alternative M_1 having high superiority value and



less inferiority value than the other alternatives. The alternative M_4 having high superiority value and less inferiority value than the alternative M_2 and M_3 . The alternative M_2 having high superiority value and less inferiority value than the alternative M_3 . Hence the alternatives are ranking as M_1, M_4, M_2 and M_3 with SR-Rule and IR-Rule.

5. Conclusion

We proposed novel MCDM method based on the hesitant Pythagorean fuzzy set. The superiority and inferiority ranking (SIR) method extended with hesitant Pythagorean fuzzy set. The proposed HPF-SIR effective results with intelligible calculation. In HPF - SIR method, we have three advantages alternatives are analyzed in the superiority and inferiority flow and experts preference values handled in a single element and intelligible solution space of Pythagorean fuzzy set. By using our proposed methods alternatives are ranked as SINA, BABA, BIDU and NETS.

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