Malaya Journal of Matematik, Vol. S, No. 1, 22-26, 2020

https://doi.org/10.26637/MJM0S20/0005

On the planarity of the inclusion ideal graph of a commutative ring

S. Kavitha[1]* and G. Gold Belsi[2]

Abstract

The *inclusion ideal graph* of a commutative ring R is a graph with vertex set Ψ where $\Psi = \{I : 0 \neq I \subset R$, where I is an ideal of R and two distinct ideals $I, J \in \Psi$ are adjacent if and only if $I \subset J$ or $J \subset I$ and it is expressed as $In(R)$. In this paper, we list out all finite commutative rings whose $In(R)$ is a path, a cycle or a tree. In addition, we classify all finite commutative rings for which the inclusion ideal graph is outer planar or planar.

Keywords

Inclusion ideal graph, unicyclic, planar.

AMS Subject Classification

13A15, 13M05, 05C25, 05C75.

[1] *Department of Mathematics, Gobi Arts and Science College, Gobichettipalayam-638476, Tamil Nadu, India.*

[2] *Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.*

***Corresponding author**: [1] kavithaashmi@gmail.com; [2] goldbelsi@gmail.com

Article History: Received **01** January **2020**; Accepted **12** February **2020**

Contents

1. Introduction

Many fascinating researches have been done in learning the structure and properties of graphs built from algebraic structures. It was first initiated by I. Beck [6], by introducing the zero- divisor graph. Thenceforth many researchers invented so many graphs from groups, rings and vector spaces (see [5, 10, 15]). Many of the properties of a ring are related to the behavior of its ideals. Besides, ideals are intensively involved in the study of ring constructions, (see [13]). Hence it is exciting to associate graphs to ideals of a ring. Using the inclusion relation in the above algebraic structure, S. Akbari, M. Habibi, A. Majidinya and R. Manaviyat [1] initiated the study of inclusion ideal graph, which is defined as follows: For a ring R with unity, the *inclusion ideal graph*, denoted by $In(R)$ whose vertex set is Ψ where $\Psi = \{I : 0 \neq I \subset R$, where I is an ideal of R and two distinct ideals $I, J \in \Psi$ are adjacent if and only if $I \subset J$ or $J \subset I$. They studied the basic properties such as connectedness, girth, clique number, chromatic number of $In(R)$ for a matrix ring and we are interesting in finding the structure of $In(R)$ for a commutative ring.

A simple graph G with n vertices for which any pair of distinct vertices is connected by the edge is called a complete graph, denoted by K_n. A complete r-partite graph is one in which the vertex set can be partitioned into r subsets and each vertex in one subset is joined to every vertex that is not in the same subset. The complete bipartite graph (2-partite graph) with part sizes m and n is notated by $K_{m,n}$. A walk is a finite or infinite sequence of edges which connects a sequence of vertices. A trail is a walk in which all edges are different. A *path* is a trail in which all vertices are distinct. A path on n vertices is denoted by P_n. A cycle in a graph G is a closed path. The *girth* of a graph G is the length of its shortest cycle and it is called by $gr(G)$. If G has no cycles then $gr(G) = \infty$. A connected graph without cycles is called a *tree*. A graph is called *unicyclic* if and only if it has exactly one cycle. For other graph theory definitions, one can refer [9].

In this paper, we deeply absorb the planarity and outer planarity of $In(R)$ and discuss some of its basic properties, for a finite commutative ring .

First let us know some theorems which are required for the posterior sections.

Theorem 1.1. [8, Theorem 2.4.1] A graph G is planar if and only if neither $K_{3,3}$ nor K_5 is a minor of G.

Theorem 1.2. [16, Theorem 1] A graph G is outer planar if and only if it has no subgraph isomorphic to $K_{2,3}$ or K_4.

2. Some Basic Properties of $In(R)$

In this section we investigate, when $In(R)$ is a path, a cycle or a tree. Also we discover the girth of $In(R)$.

Note 2.1. *Suppose R is a field, then $In(R)$ does not exist.*

Suppose $R = \mathbb{Z}_{p^n}$, where p is a prime and n is a positive integer and $n \geq 2$, then $In(R)$ is complete.

Proof. Let $\mathfrak{m}$ be the maximal ideal of R. Then clearly we have the following sequence $I_1 \subset I_2 \subset \ldots I_n \subset \mathfrak{m}$, where $I_i, 1 \leq i \leq n$ are the distinct non-trivial ideals of R. Hence we get, $In(R) \cong K_{p^{n+1}}$. □

Theorem 2.2. *Let R be a finite commutative non-local ring with unity. Then $gr(In(R)) \in \{3, 6, \infty\}$.*

Proof. We know that any finite commutative ring R can be written as $R = R_1 \times \cdots \times R_n$, where each $(R_i, \mathfrak{m}_i)$ is a local ring. Assume that $n \geq 4$. Let $x_1 = R_1 \times R_2 \times R_3 \times 0 \times \ldots \times 0, x_2 = R_1 \times R_2 \times 0 \times \ldots \times 0, x_3 = R_1 \times 0 \times R_3 \times 0 \times \ldots \times 0 \in V(In(R))$. Then $x_1 - x_2 - x_3 - x_1$ is a cycle in $In(R)$ and hence $gr(In(R)) = 3$.

Assume that $n = 3$. If R_i is a field for all $i = 1, 2, 3$, then $In(R) \cong C_6$ and hence $gr(In(R)) = 6$. Suppose R_1 is not a field, the $\mathfrak{m}_1 \neq 0$ and so $\mathfrak{m}_1 \times R_2 \times R_3 - \mathfrak{m}_1 \times 0 \times R_3 - \mathfrak{m}_1 \times 0 \times 0 - \mathfrak{m}_1 \times R_2 \times R_3$ is a cycle in $In(R)$ and hence $gr(In(R)) = 3$.
Assume that $n = 2$. Suppose $\mathfrak{m}_i \neq 0$ for all i, then $\mathfrak{m}_1 \times R_2 - \mathfrak{m}_1 \times 0 - \mathfrak{m}_1 \times \mathfrak{m}_2 - \mathfrak{m}_1 \times R_2$ is a cycle in $In(R)$ and hence $gr(In(R)) = 3$. Suppose $\mathfrak{m}_i = 0$ for all i, then $In(R) \cong \overline{K_2}$ and hence $gr(In(R)) = \infty$. Suppose $\mathfrak{m}_1 \neq 0$ and $\mathfrak{m}_2 = 0$. If R_1 contain at least two nontrivial ideals, say I and $\mathfrak{m}_1$, then $\mathfrak{m}_1 \times R_2 - \mathfrak{m}_1 \times 0 - I \times 0 - \mathfrak{m}_1 \times R_2$ is a cycle in $In(R)$ and hence $gr(In(R)) = 3$. If R_1 contain only one nontrivial ideal, then $In(R) \cong P_4$ and hence $gr(In(R)) = \infty$. □

Theorem 2.3. *Let R be a finite commutative non- local ring with unity. Then $In(R)$ is a path if and only if $R = R_1 \times F_1$ where $(R_1, \mathfrak{m}_1$ is local and F_1 is a field.*

Proof. Suppose that $R = R_1 \times F_1$ where R_1 is a local ring with $\mathfrak{m}_1$ as the only nontrivial ideal and F_1 is a field, then $In(R) \cong P_4$.

Conversely, suppose $In(R)$ is a path. Then $gr(In(R)) = \infty$. We know that $R = R_1 \times \cdots \times R_n$, where each $(R_i, \mathfrak{m}_i)$ is local and $n \geq 2$. If $n \geq 3$, then by the proof of Theorem 2.2, $gr(In(R)) = 3$ or 6, which produces a contradiction. When $n = 2$, using Theorem 2.2, we get that $R = R_1 \times F_1$ where R_1 is a local ring with $\mathfrak{m}_1$ as the only nontrivial ideal and F_1 is a field. □

Theorem 2.4. *Let R be a finite commutative non- local ring with unity. Then $In(R)$ is a cycle if and only if $R = F_1 \times F_2 \times F_3$, where each F_i is a field.*

Proof. The proof is apparent by Theorem 2.2 □

Theorem 2.5. *Let R be a finite commutative non- local ring with unity. Then $In(R)$ is a tree if and only if $R = R_1 \times F_1$ where $R_1, \mathfrak{m}_1$ is local with $\mathfrak{m}_1$ as the only nontrivial ideal and F_1 is a field.*

Proof. The proof is straightforward using Theorems 2.2, 2.3 and 2.4. □

Theorem 2.6. *Let R be a finite commutative non- local ring with unity. Then $In(R)$ is unicyclic if and only if $R = F_1 \times F_2 \times F_3$, where each F_i is a field.*

Proof. Suppose $In(R)$ is unicyclic. We have $R \cong R_1 \times \cdots \times R_n$, where each $(R_i, \mathfrak{m}_i)$ is local and $n \geq 2$. Suppose $n \geq 4$. Let $y_1 = R_1 \times R_2 \times R_3 \times 0 \times \ldots \times 0, y_2 = R_1 \times R_2 \times 0 \times \ldots \times 0, y_3 = R_1 \times 0 \times 0 \times \ldots \times 0, y_4 = R_1 \times R_2 \times 0 \times R_4 \times 0 \times \ldots \times 0, y_5 = R_1 \times 0 \times 0 \times R_4 \times 0 \times \ldots \times 0 \in V(In(R))$. Then $In(R)$ has two cycles, $y_1 - y_2 - y_3 - y_1$ and $y_4 - y_5 - y_3 - y_4$, which is a contradiction. Hence $n \leq 3$.

Let $R = R_1 \times R_2$. Suppose $\mathfrak{m}_i \neq 0$ for all i. Then $R_1 \times \mathfrak{m}_2 - R_1 \times 0 - \mathfrak{m}_1 \times 0 - R_1 \times \mathfrak{m}_2$ and $R_1 \times \mathfrak{m}_2 - \mathfrak{m}_1 \times \mathfrak{m}_2 - \mathfrak{m}_1 \times 0 - R_1 \times \mathfrak{m}_2$ are two cycles in $In(R)$, a contradiction. Hence $\mathfrak{m}_i = 0$ for some i. Without loss of generality, assume that R_2 is a field. Since $In(R)$ is unicyclic, by Theorem 2.3, R_2 is not a field. Suppose R_1 contains at least two nontrivial ideals, say I and $\mathfrak{m}_1$, then $R_1 \times 0 - \mathfrak{m}_1 \times 0 - I \times 0 - R_1 \times 0$ and $\mathfrak{m}_1 \times R_2 - \mathfrak{m}_1 \times 0 - I \times 0 - \mathfrak{m}_1 \times R_2$ are two cycles in $In(R)$, a contradiction. If R_1 contains exactly one nontrivial ideal, then by Theorem 2.3, $In(R) \cong P_4$, a contradiction.
Assume that $n = 3$. If $m_1 \neq 0$, then $R_1 \times R_2 \times 0 - \mathfrak{m}_1 \times R_2 \times 0 - 0 \times R_2 \times 0 - R_1 \times R_2 \times 0$ and $R_1 \times R_2 \times 0 - R_1 \times 0 \times 0 - \mathfrak{m}_1 \times 0 \times 0 - R_1 \times R_2 \times 0$ are two cycles in $In(R)$, a contradiction. Hence $R = F_1 \times F_2 \times F_3$. The converse follows from Theorem 2.4. □

3. Outer planarity of $In(R)$

An outer planar graph is a graph which can be drawn in the plane without any intersections, in such a way that all vertices belong to the outer face of the embedding. Let us characterize all finite commutative rings for which $In(R)$ is outer planar. In the posterior sections we notate, k_i as the number of distinct nontrivial ideals of R_i.

Theorem 3.1. *$R = R_1 \times \cdots \times R_n$ be a commutative non- local ring with identity, where each $(R_i, \mathfrak{m}_i)$ is a local ring. Then $In(R)$ is not outer planar for $n \geq 2$.*

Proof. Let $n \geq 2$ with $k_i = 1$ for $1 \leq i \leq n$, then the subgraph induced by the set $\{\mathfrak{m}_1 \times 0 \times \ldots \times 0, 0 \times \mathfrak{m}_2 \times 0 \times \ldots \times 0, R_1 \times \mathfrak{m}_2 \times 0 \times \ldots \times 0, \mathfrak{m}_1 \times R_2 \times 0 \times \ldots \times 0, \mathfrak{m}_1 \times \mathfrak{m}_2 \times 0 \times \ldots \times 0\}$ contains $K_{2,3}$. By Theorem 1.2, we get a contradiction. Hence the theorem. □

Theorem 3.2. *Let $R = F_1 \times \cdots \times F_n$, where each F_i is a finite field and $n \geq 2$. Then $In(R)$ is outer planar if and only if $n \leq 3$.*

Proof. We know that, $In(F_1 \times F_2 \times F_3) \cong C_6$, which is outer planar. Suppose $n \geq 4$, then the subgraph induced by the set $\{F_1 \times 0 \times \ldots \times 0, 0 \times F_2 \times 0 \times \ldots \times 0, F_1 \times F_2 \times 0 \times \ldots \times 0, F_1 \times F_2 \times F_3 \times 0 \times \ldots \times 0, F_1 \times F_2 \times 0 \times F_4 \times 0 \times \ldots \times 0\}$ contains $K_{2,3}$. By Theorem 1.2, we get a contradiction. □

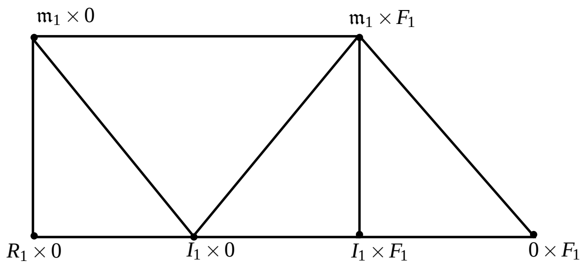

Figure. 1. $In(R_1 \times F_1)$ with $k_1 = 2$

Theorem 3.3. *Let $R = R_1 \times \cdots \times R_n \times F_1 \times \cdots \times F_m$, where each $(R_i, \mathfrak{m}_i)$ is a local ring, F_j is a field and $m, n \geq 1$ and R_i, F_j are finite. Then $In(R)$ is outer planar if and only if $m = n = 1$ and $k_1 \leq 2$.*

Proof. When $n = m = k_1 = 1$, $In(R) \cong P_4$. Also when $n = m = 1$ and $k_1 = 2$, $In(R)$ is figure.1 , which are outer planar. Suppose $n = m = 1$ and $k_1 \geq 3$, then the subgraph induced by the set $\{0 \times F_1 \times \ldots \times 0, I_2 \times F_1 \times 0 \times \ldots \times 0, I_1 \times F_1 \times 0 \times \ldots \times 0, \mathfrak{m}_1 \times F_1 \times 0 \times \ldots \times 0, I_2 \times 0 \times \ldots \times 0\}$ contains $K_{2,3}$. Using Theorem 1.2,we get a contradiction. Also when $n = k_1 = 1$ and $m \geq 2$, the following set, $\{\mathfrak{m}_1 \times 0 \times \ldots \times 0, 0 \times F_1 \times 0 \times \ldots \times 0, \mathfrak{m}_1 \times F_1 \times 0 \times \ldots \times 0, R_1 \times F_1 \times 0 \times \ldots \times 0, \mathfrak{m}_1 \times F_1 \times F_2 \times 0 \times \ldots \times 0\}$ yields $K_{2,3}$ as a subgraph in $In(R)$. Here also we get a contradiction using Theorem 1.2. Also when $n \geq 2$, by Theorem 3.1, $In(R)$ is not outer planar. Hence the theorem is proved. □

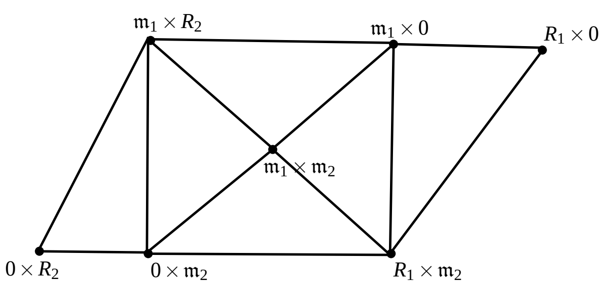

Figure. 2. $In(R_1 \times R_2)$ with $k_1 = k_2 = 1$

4. Planarity of $In(R)$

A graph is planar, if it can be embedded in the plane without any crossings. Since every outer planar graph is planar, in this section we will distinguish $In(R)$ which are planar, but not outer planar.

Theorem 4.1. *$R = R_1 \times \cdots \times R_n$ be a ring with identity, where each $(R_i, \mathfrak{m}_i)$ is a finite local ring. Then $In(R)$ is planar if and only if $n = 2$ and $k_1 = k_2 = 1$.*

Proof. Assume that $In(R)$ is planar. Suppose that $n \geq 3$. Let $\Omega_1 = \{\mathfrak{m}_1 \times R_2 \times \ldots R_n, R_1 \times \mathfrak{m}_2 \times R_3 \times \ldots \times R_n, R_1 \times R_2 \times \mathfrak{m}_3 \times R_4 \times \ldots \times R_n, \mathfrak{m}_1 \times 0 \times 0 \times \ldots \times 0, 0 \times \mathfrak{m}_2 \times 0 \times \ldots \times 0, 0 \times 0 \times \mathfrak{m}_3 \times 0 \times \ldots \times 0\} \subset V(In(R))$. Then the $\langle \Omega_1 \rangle$ in $In(R)$ contains $K_{3,3}$ as a minor. By Theorem 1.1, we obtain a contradiction. Hence $n = 2$.

Suppose $k_1 \geq 2$. Let the ideals be $I_1 \subset \mathfrak{m}_1$ and $\Omega_2 = \{\mathfrak{m}_1 \times R_2, R_1 \times \mathfrak{m}_2, \mathfrak{m}_1 \times \mathfrak{m}_2, \mathfrak{m}_1 \times 0, 0 \times \mathfrak{m}_2, I_1 \times 0\} \subset V(In(R))$. Then $\langle \Omega_2 \rangle$ in $In(R)$ contains $K_{3,3}$ as a subgraph. By Theorem 1.1, we obtain a contradiction. Hence $k_1 = 1$ and $k_2 = 1$. The converse follows from figure. 2.

□

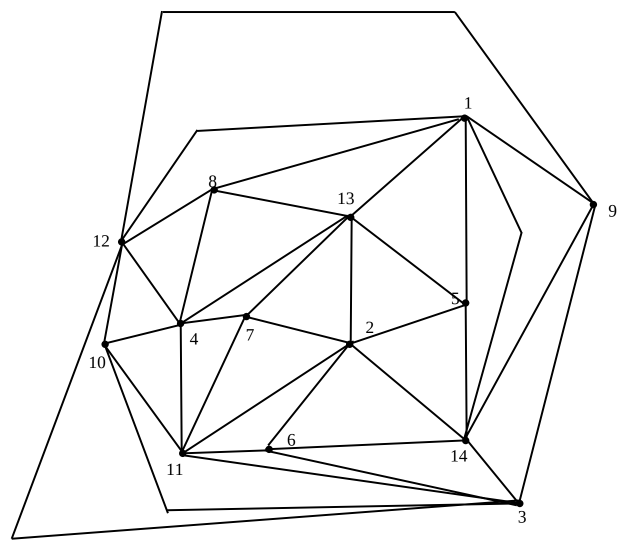

Figure. 3. $In(F_1 \times F_2 \times F_3 \times F_4)$

In figure. 3, we replace $F_1 \times 0 \times 0 \times 0$ by 1, $0 \times F_2 \times 0 \times 0$ by 2, $0 \times 0 \times F_3 \times 0$ by 3, $0 \times 0 \times 0 \times F_4$ by 4, $F_1 \times F_2 \times 0 \times 0$ by 5, $0 \times F_2 \times F_3 \times 0$ by 6, $0 \times F_2 \times 0 \times F_4$ by 7, $F_1 \times 0 \times 0 \times F_4$ by 8, $F_1 \times 0 \times F_3 \times 0$ by 9, $0 \times 0 \times F_3 \times F_4$ by 10, $0 \times F_2 \times F_3 \times F_4$ by 11, $F_1 \times 0 \times F_3 \times F_4$ by 12, $F_1 \times F_2 \times 0 \times F_4$ by 13 and $F_1 \times F_2 \times F_3 \times 0$ by 14.

Theorem 4.2. *Let $R = F_1 \times \cdots \times F_n$, where each F_i is a finite field and $n \geq 2$. Then $In(R)$ is planar if and only if $n = 4$.*

Proof. Assume $In(R)$ is planar. When $n \geq 5$, the subgraph induced by the set $\{F_1 \times 0 \times \ldots \times 0 \times 0, 0 \times F_2 \times 0 \times \ldots \times 0, 0 \times 0 \times F_3 \times 0 \times \ldots \times 0, F_1 \times F_2 \times F_3 \times 0 \times \ldots \times 0, F_1 \times F_2 \times F_3 \times F_4 \times 0 \times \ldots \times 0, F_1 \times F_2 \times F_3 \times 0 \times F_5 \times 0 \times \ldots \times 0\}$ contains $K_{3,3}$. Then we get a contradiction using Theorem 1.1. Hence by Theorem 3.2, $n = 4$. The converse follows from figure. 3. □

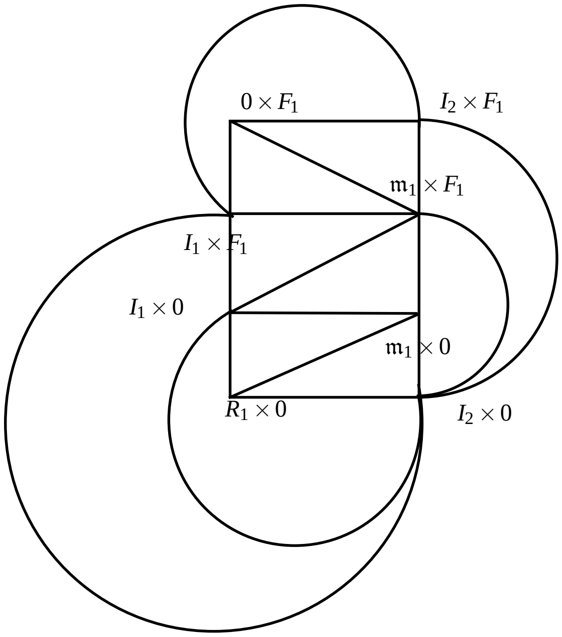

Figure. 4. $In(R_1 \times F_1)$ with $k_1 = 3$

Theorem 4.3. *Let $R = R_1 \times \cdots \times R_n \times F_1 \times \cdots \times F_m$ be a ring with identity, where each $(R_i, \mathfrak{m}_i)$ is a finite local ring, F_j is a finite field and $m, n \geq 1$. Then $In(R)$ is planar if and only if $n = k_1 = 1, m = 2$ or $m = n = 1$ and $k_1 = 3$.*

Proof. **Case 1.** $n \geq 2$ with $k_i = 1$ for all i.
Let $A = \{1, 2, \ldots 6\}$ where $1 = 0 \times 0 \times F_1 \times 0 \ldots \times 0, 2 = \mathfrak{m}_1 \times 0 \times \ldots \times 0, 3 = 0 \times \mathfrak{m}_2 \times 0 \times \ldots \times 0, 4 = \mathfrak{m}_1 \times 0 \times F_1 \times 0 \times \ldots \times 0, 5 = R_1 \times \mathfrak{m}_2 \times F_1 \times 0 \times \ldots \times 0, 6 = \mathfrak{m}_1 \times \mathfrak{m}_2 \times F_1 \times 0 \times \ldots \times 0$. Then the subgraph $\langle A \rangle$ contains $K_{3,3}$. Then by Theorem 1.1, $g(In(R))$ is non- planar.
Hence $n = 1$.
Case 2. $m \geq 3$ with $k_1 = 1$.
Let $B = \{1, 2, \ldots 6\}$ where $1 = \mathfrak{m}_1 \times F_1 \times F_2 \times F_3 \times 0 \times \ldots \times 0, 2 = R_1 \times F_1 \times F_2 \times 0 \ldots \times 0, 3 = 0 \times F_1 \times 0 \times \ldots \times 0, 4 = 0 \times 0 \times F_2 \times \ldots \times 0, 5 = \mathfrak{m}_1 \times 0 \times 0 \times \ldots \times 0, 6 = \mathfrak{m}_1 \times F_1 \times F_2 \times 0 \times \ldots \times 0$. Then the subgraph of $\langle B \rangle$ contains $K_{3,3}$. Then by Theorem 1.1, $g(In(R))$ is non- planar.
Hence $m \leq 2$.
Then $R = R_1 \times F_1$ or $R_1 \times F_1 \times F_2$.
Suppose $R = R_1 \times F_1 \times F_2$.
Suppose $k_1 \geq 2$. Let the ideals be, $I_n \subset \ldots I_1 \subset \mathfrak{m}_1$. Let $D = \{1, 2, \ldots 6\}$ where $1 = 0 \times F_1 \times \ldots \times 0, 2 = R_1 \times F_1 \times 0 \times \ldots \times 0, 3 = \mathfrak{m}_1 \times 0 \times \ldots \times 0, 4 = \mathfrak{m}_1 \times F_1 \times 0 \times \ldots \times 0, 5 = I_1 \times 0 \times \ldots \times 0, 6 = I_1 \times F_1 \times 0 \times \ldots \times 0$. Then $\langle D \rangle$ has $K_{3,3}$ as a subgraph. Then by Theorem 1.1, we receive a contradiction.
Hence $k_1 = 1$ and the converse follows from figure. 5.
Suppose $R = R_1 \times F_1$ and $k_1 \geq 4$. Let the ideals be $I_n \subset \ldots \subset I_3 \subset I_2 \subset I_1 \subset \mathfrak{m}_1$. Let $G = \{1, 2, \ldots 6\}$ where $1 = I_2 \times 0 \times \ldots \times 0, 2 = I_3 \times 0 \ldots \times 0, 3 = 0 \times F_1 \times 0 \times \ldots \times 0, 4 = \mathfrak{m}_1 \times F_1 \times 0 \times \ldots \times 0, 5 = I_1 \times F_1 \times 0 \times \ldots \times 0, 6 = I_2 \times F_1 \times 0 \times \ldots \times 0$. Then a subgraph of $\langle G \rangle$ contains $K_{3,3}$. By Theorem 1.1, we have a contradiction.

Now by Theorem 3.3, $k_1 = 3$ and the converse follows from figure. 4. $\square$

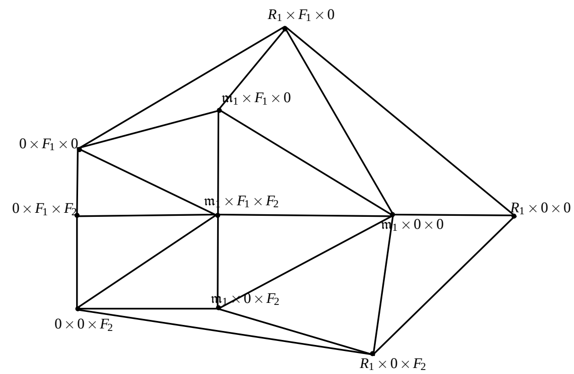

Figure. 5. $In(R_1 \times F_1 \times F_2)$ with $k_1 = 1$

5. Acknowledgement

This research work is supported by MANF programme (201718- MANF- 2017 -18- TAM- 82372) of UGC, Government of India for the second author.

References

[1] S. Akbari, M. Habibi, A. Majidinya and R. Manaviyat, The inclusion ideal graph of rings, *Comm. Algebra*, 43 (6)(2015), 2457 – 2465.

[2] S. Akbari, M. Habibi, A. Majidinya and R. Manaviyat, On the inclusion ideal graph of rings, *Electron. Notes Discrete Math.*, 45(2014), 73–78 .

[3] Angsuman Das, Subspace inclusion graph of a vector space, *Comm. Algebra*, 44(2016), 554-564.

[4] M.F. Athiya and I.G.Macdonald, *Introduction to Commutative Algebra*, Anderson - Wiley Publishing Company , California, (1969).

[5] A. Badawi, On the annihilator graph of a commutative ring, *Comm. Algebra*, 42 (1) (2014), 108-121.

[6] I. Beck, Coloring of commutative rings, *J. Algebra*, 116 (1988), 208-226.

[7] R. Belshoff and J. Chapman, Planar zero- divisor graphs, *J. Algebra*, 316 (1)(2007), 471-480.

[8] Bojan Mohar and Carsten Thomassen, *Graphs on Surfaces*, The Johns Hopkins University Press, Baltimore and London, (1956).

[9] J. A. Bondy and U.S.R.Murty, *Graph Theory and its Applications*, American Elsevier, Newyork, (1976).

[10] Chanlemki Lanong and Sanghita Dutta, Some results on graphs associated with vector spaces, *J. Inf. Opp. Sciences*, 38 (2017), 1357– 1368.

[11] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Second edition, Wiley Student edition, New Jersey, (2008).

[12] N. Jahanbakhsh, R. Nikandish, M. Nikmehr, On the inclusion ideal graph of a poset, *Algebra Discrete Math.*, 27(2019), 269–279.

[13] A. V. Kelarev and D.S. Passman, A description of incidence rings of group automata, *Contemporary Mathematics*, 456 (2008), 27–33.

[14] M. Ghebleh, M. Khatirinejad, Planarity of iterated line graphs, *Discrete Math.*, 308 (2008), 144–147.

[15] X. Ma, H. Wei and L. Yang, The Coprime graph of a group,*Int. J. Group Theory*, 3 (2014), 13–23.

[16] Maciej M. Syslo, Characterizations of Outer planar graphs, *Discrete Math.*, 26 (1979), 47– 53.

[17] S. P. Redmond, On zero- divisor graphs of small finite commutative rings, *Discrete Math.*, 307 (2007), 1155–1166.

[18] A. T. White, *Graphs, Groups and Surfaces*, North-Holland, Amsterdam, (1984).

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆