



# Analytical solution of non linear reaction equation in the spruce budworm and forest

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## Abstract

In this paper, we are presenting the investigation of reaction in the spruce budworm and forest by solving non linear time dependent equation with boundary circumstances with the help of Variational iteration method.

## Keywords

Ethnomathematics, Non linear equation, Modelling, Differential equations, Numerical methods, Boundary value problem.

## AMS Subject Classification

00A71, 03C65, 32G13, 34A25, 34A35, 34B15.

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## 1. Introduction

A structure of differential equations is claiming to be a nonlinear, if it is not a linear system. Problems connecting nonlinear differential equations are tremendously diverse, and methods of solution dependent. Examples of nonlinear differential equations are the Navier–Stokes equations in fluid dynamics and therefore the Lotka–Volterra equations in biology.

One of the best involvedness of nonlinear problems is that it is not in the main possible to mix existing solutions into new-fangled solutions. In the linear problems, for instance, a ancestors of linearly independent solutions are often wont to knock down general solutions through the principle of superposition . an candid example of this is often one-dimensional heat transport with Dirichlet boundary conditions, the answer

of which may be written as a time-dependent linear mixture of sinusoids of differing frequencies; this makes solutions very flexible. it's often possible to seek out numerous very specific solutions to nonlinear equations, however the shortage of a principle of superposition prevents the development of latest solutions.

Expository strategies ordinarily used to explain nonlinear conditions are limited and numerical methods including discretization of the factors then again offer ascent to adjusting blunders. This technique is ideal over numerical strategies as it is liberated from adjusting blunders and neither requires huge PC power/memory. He has applied this technique for acquiring systematic arrangements of independent standard differential condition, nonlinear halfway differential conditions with variable coefficients, and integro-differential conditions. The variational emphasis technique was effectively utilized by different creators. For instance, the VIM was applied to the nonlinear Boltzmann condition [3], to Burger's and coupled Burger's conditions [4], to the eikonal incomplete differential condition [5], to explanatory integro-differential conditions emerging in heat conduction in materials with memory [6], to coupled Korteweg-de Vries (KdV) and Boussinesq-like B(m,n) conditions [7], to Sawada-Kotera conditions [8], to changed Camassa-Holm and Degasperis-Procesi conditions [9], to KdV, K(2,2), Burgers, and cubic Boussinesq conditions [10] and to KdV-Burgers and Sharma-TassoOlver conditions [11].

## 2. Mathematical Formulation of the problem

In order to express the earlier, we pick a logistic form: if  $B$  represents the budworm solidity, then in the nonattendance of predation  $B$  satisfies

$$\frac{dB}{dt} = r_B B \left(1 - \frac{B}{K_B}\right) \tag{2.1}$$

Where  $r_B$  is the central budworm growth rate,  $B$  is the budworm density and  $k_B$  is the carrying capacity unspecified to attend ahead the amount of undergrowth available. The logistic equation is chosen here for the reason that it involves only two parameter. The later mathematics analysis is facilitated by this choice, but the results would be analogous if any other form of self-limited growth were assumed.

We conclude that  $g(B)$  should approach an upper limit  $\beta$  as  $\beta \rightarrow \infty$ . This limit  $\beta$  may depend upon the show variables (i.e. the forest variables). Thus we may assume that  $g(B)$  vanishes quadratically as  $\beta \rightarrow 0$ . A opportune form for  $g(B)$  which has the properties of infiltration at a level  $\beta$  and  $\beta^2$  which vanishes like is

$$g(B) = \beta \left(\frac{B^2}{\alpha^2 + B^2}\right) \tag{2.2}$$

Where  $\beta$  is the maximum budworm predated,  $\alpha$  is the maximum density for the predation,  $B$  is the budworm density. The parameter  $\alpha$  in Eq.(2.2) determines the scale of budworm densities at which saturation begins to take place. The addition of vertebrate predation to Eq.(2.1) thus produces a total equation for the rate of change of dimensional budworm density  $B$ :

$$\frac{dB}{dt} = r_B B \left(1 - \frac{B}{K_B}\right) - \beta \left(\frac{B^2}{\alpha^2 + B^2}\right) \tag{2.3}$$

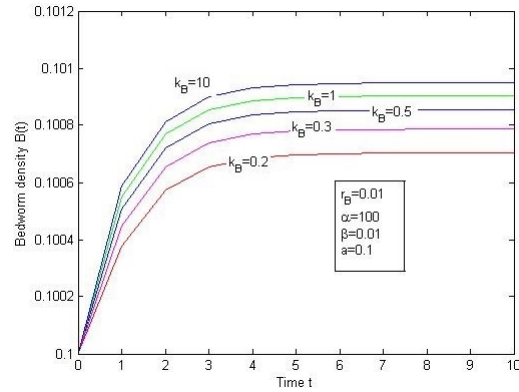
Where accentuate that this exacting form was chosen to require as few parameters as possible: our final termination are not dependent upon the specific form of the equation, but only upon its qualitative properties. The initial conditions are

$$t = 0, B = a \tag{2.4}$$

where  $r_B, K_B, \beta$  and  $\alpha$  the reaction parameters. The later expressions must be solved to the boundary conditions presented in the Eq. (2.4). Since the equations are non-linear because of the Michaelis-Menten kineic term [5-2], it will be a problematic to obtain a fully rigorous solution to the later. However, we will develop an exact solution for the latter expressions in our future work.

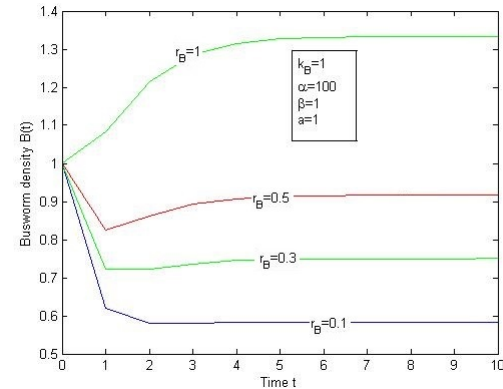
## 3. Analytical Solution of Bedworm Density by Variational Iteration Method

Recently many authors have applied the VIM (Ghori, Amed and Siddiqui (2007)), the above system of non-linear



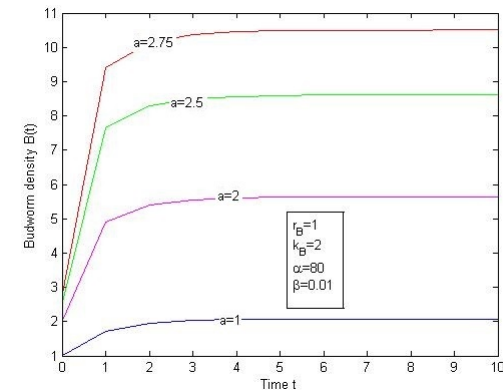
**Figure 1**

Fig.1.The normalized budworm density of  $B(t)$  is plotted. The profile was computed using Equation (5) for the different value of  $k_B$ , when  $\gamma_B = 0.01, \alpha = 0.1, \beta = 0.01, a = 0.1$



**Figure 2**

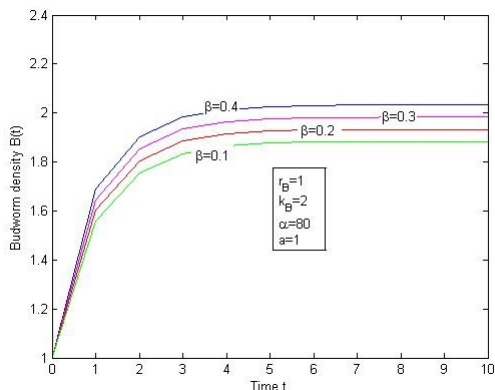
Fig.2.The normalized budworm density of  $B(t)$  is plotted. The profile was computed using Equation (5) for the different value off  $\gamma_B$ , when  $k_B = 1, \alpha = 0.1, \beta = 100, a = 1$



**Figure 3**

Fig.3.The normalized budworm density of  $B(t)$  is plotted. The profile was computed using Equation (5) for the different value of  $a$  when  $\gamma_B = 1, k_B = 2, \alpha = 80, \beta = 0.01$





**Figure 4**

Fig.4.The normalized budworm density of B(t) is plotted. The profile was computed using Equation (5) for the different value of  $\beta$  when  $\gamma_B = 1, k_B = 2, \alpha = 80, a = 1$

equations can be solved analytically in a simple and closed form using Variational iteration method (Ref. Appendix-2.A). the solution of the above Eq. (3) becomes,

$$B_1(t) = \begin{cases} a + \frac{a}{\alpha^2} \left[ \left( \frac{a^2 e^{3t}}{-t} \right) + \frac{a^3}{3} \right] + r_B [-a e^{-3t} + a] \\ - \frac{r_B}{K_B} \left( \frac{a^2 e^{-2t}}{-2} + \frac{a^2}{2} \right) + \frac{r_B}{\alpha^2} \left( \frac{a^3 e^{-3t}}{-3} + \frac{a^3}{3} \right) \\ - \frac{r_B}{K_B \alpha^2} \left( \frac{a^4 e^{-4t}}{-4} + \frac{a^4}{4} \right) \\ - \frac{\beta}{\alpha^2} \left( \frac{a^2 e^{-2t}}{-2} + \frac{a^2}{2} \right) \end{cases} \quad (3.1)$$

### 4. Discussion

Equation (3.1) represents the novel approximate diagnostic expression of budworm density for all standards of the parameter. Figures 1- 4 are drawn for various values of the rate constants of the budworm density. From these figures, it is inferred that the value of budworm density increases when the value  $k_B$  increases. The budworm density is slowly increasing and abruptly reaches the unvarying value when  $t \geq 2$  for all values of a.

### 5. Conclusion

In this paper, the time self-determining nonlinear reaction equation has been prepared and solved analytically using variational iteration method. We have presented a simple and closed form of an analytical expressions corresponding to the budworm density  $B(t)$  for all values of  $\alpha$ . Moreover, on the basis of the conclusion of this work, it is possible to calculate the approximate amounts of the budworm density. The extension of the procedure to other two-dimensional geometries with various complex boundary conditions seems possible.

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