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Graph coloring and labeling applications in computer science

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Abstract

Graphs theory plays a vital role in various fields. Graph concepts are used to model many types of relations and processes in physical, biological, social and information systems. The use of graph theory concepts is also seen in different research areas of computer science like Network, Database management system, Artificial intelligence, Software architecture, Design of algorithms, Multiprocessing, Data structure, image processing etc. This paper mainly concentrates on applications that uses graph coloring and graph labeling concept.

Keywords

Four color theorem, Map coloring, Graph labeling, and Communication network.

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Contents

1. Introduction

To study and model various application in different fields Graph theoretical concepts are widely used. In computer science to model pair wise relations between objects from certain collection the mathematical structure called graphs are used. In data structures graphs are represented as trees making use of vertices and edges. Path, walk, circuit, cut sets, coloring, trees, algorithms and other concepts of graph theory are used operating system, software engineering, image processing, communication networks, and in data mining. Graph coloring is mainly used in research fields of computer science like networking, data mining, image processing etc.

Modeling of network topologies, data base design, scheduling, travelling salesman problem, guarding art gallery are some of the applications that use graph coloring concept. Graph coloring is assigning colors to the vertices or edges of graph so that no two adjacent vertices or edges will have same color. Graph coloring should be done using minimum number of colors called the chromatic number. Graph colored using minimum number of colors is called properly colored graph.

Labeling of graphs is used in heterogeneous fields. Some applications like communication network addressing, designing fault tolerant system, automatic channel allocation uses graph labeling.Graph is called a labeled graph if each edge $e = UV$ is given the value $f(UV) = f(u) * f(v)$, where $*$ is a binary operation such as addition, multiplication, modulo addition or absolute difference, modulo subtraction or symmetric difference. Massive graphs are used to represent the global data structures available in social, communication networks to World Wide Web. These graphs help to retrieve or access the global data easily.

2. Applications of graph coloring

(i) Map coloring in GSM networks In Mobile phone network like Groups Special Mobile (GSM) the geographical area of the network is divided into hexagonal regions called cells. Mobile phones within the cell will be connected to communication tower that exist in that cell. Mobile phones connect to the GSM network by searching for cells in the neighbors. Four color theory is used in GSM Since it operates only in four different frequency ranges. The cellular regions are properly colored using four colors.

The vertex coloring algorithm can also be used to assign at most four different frequencies for any GSM mobile phone network [7]. Map drawn on the plane uses a four color theorem to color the regions of a map properly using at most four distinct colors. The proper coloring should be such that no two adjacent regions are assigned the same color. For the given map we construct the dual graph. The dual graph is

constructed by placing vertex inside each region of a map. The vertex inside two regions will be connected by an edge if those two regions have a common edge forming the boundary.

The chromatic number of the dual graph gives the chromatic number of the original map. The vertex coloring of dual graph is same as the region coloring of the map. So the four color theory can be used in GSM network where four colors are the four frequency ranges in which the network operates. Figure 3 shows the map of India colored using four colors. Figure 4 shows the dual graph constructed for the map. The vertex coloring is used for the dual graph. Maximum of four colors are used for vertex coloring hence map coloring also requires maximum of four colors. So in GSM network four color theorem can be used to assign four frequencies [2].

(ii). Aircraft Scheduling Graph coloring can be used to schedule k aircrafts that has to assign to n flights. The ith flight is at the time interval (a_i, b_i). We cannot assign the aircraft if the two flight overlap. The interval graph used in this case uses nodes that represent flights. The two nodes are connected by an edge if the time intervals overlap. The interval graph can be colored optimally in polynomial time

(iii) Biprocessor Tasks Assume that there is a set of processors and a set of tasks. A processor cannot work on two jobs at the same time. If there is a task that has to be executed on two processors like in scheduling file transfer between processors graph coloring can be used. The graph constructed in this case represents a node that corresponds to a processor. If a task has to be performed on two processor then the two nodes representing that two processors will connected by an edge. Now task allocation can be done by assigning color to the edges of the graph such that a vertex appears atmost once.

3. Applications of graph labeling

Based on the problem suitable labeling on the graph can be applied.

(i) Fast Communication in sensor networks Using Radio Labeling. Radio labeling is used in assigning a channel to each station such that interference can be avoided. Smaller the distance between the stations stronger will be the interference. To avoid interference a positive integer is assigned to each channel. Channel assignment should be stronger when distance between the stations is smaller. In radio labeling each node represent a transmitter. An edge is used connected two nodes if the two transmitters are adjacent. Radio labeling is connected graph defined as $G(V,E)$. $d(a,b)$ represents the distance between two vertices a and b in G. $diam(G)$ is the diameter of G. The maximum distance between any pair of vertices in G. A radio labeling (or multilevel distance labeling) for G is an injective function $f : V(G) \to N$ union $\{0\}$ such that for any vertices u and v, $|f(u)If(v)| >= diam(G) - d(u,v) + 1$. Radio labeling is the efficient way of determining the time of communication in sensor network [8].

(ii) Designing Fault Tolerant Systems with Facility Graphs Facility graphs are used to design fault tolerance system with specified degree. The interconnection between the computing facilities of a particular computation in shown in the form of graph called facility graph. The node of this graphrepresents system facilities (hardware/software) and edge represent access link between facilities. Control units, arithmetic processors, input/output equipment etc come under hardware facility and compilers, application programs, library routines etc come under software facilities. The real time systems are represented by facility graph since a facility can access some other facility while operating. Graph vertex labeling is used in facility graph [6].

(iii) Graph Labeling in Communication Relevant to Adhoc Networks Mobile Adhoc Networks (MANETS) issues can also be resolved using graph labeling. Graph model can be used to analyze the issues like connectivity, scalability, routing, modeling the network and simulation are to be considered. Graphs can be represented as matrices and algorithms can be used to analyze the issues. Node density, mobility among the nodes, link formation between the nodes and packet routing can be simulated using concepts random graph. The congestion in MANET's can be analyzed using various algorithms are also available to analyze where these networks are modeled based on graph theoretical ideas.

(iv) Automatic Routing with labeling A static network can be represented as a specific kind of graph by connecting nodes in some topology and labeling can be applied for automatic routing of data in a network. The graph can be cycle, path, circuit, walk, connected which represent a fixed network. For each network labeling is done with a constant which helps routing to automatically detect next node in the network [12].

4. Conclusion

The main aim of this paper is to explore role of Graph coloring and labeling in various fields of computer science. Graph coloring and Labeling is powerful tool that makes things easier. Various papers based on graph coloring andlabeling have been studied. The concept has been related to different applications in computer science and overview has been presented

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