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ESAN-ESA-A new zeros assignment method for finding optimal tour plan of traveling salesman problems

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Abstract
A traveling salesman problem (TSP) is an assignment problem (AP) with additional constraints, in which the objective is to find the best possible way of visiting all the cities and returning to the starting point that minimize the overall travel cost (or travel distance). It occupies a very significant role in the real physical world. The most common method used to solve the TSPs is the Hungarian assignment method. In this paper, we make an effort to introduce a new zeros assignment approach namely ESAN-ESA method for finding an optimal tour plan to a TSP. The proposed ESAN-ESA method is based on the principle of reducing the given cost (or distance) matrix to a reduced cost matrix (RCM) or a reduced distance matrix (RDM) having at least one zero in each row and in each column and making assignments to the optimally selected 0-entry cells of the RCM which ensures best tour plan for a given TSP. The ESAN-ESA method finds an optimal tour plan to a TSP in two phases. In the first phase, a set of optimal subtours is generated using the zeros assignment technique based on the ME rules and in the second phase linking of all the subtours together to form a complete round optimal tour is carried out based on the available 0-entry or 1-entry or the next higher entry among the upcoming unassigned cells.

Keywords
Assignment Problem, Traveling Salesman Problem, Hungarian method, ESAN-ESA method.

AMS Subject Classification
90C08, 90C00.

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Contents

1. Introduction

The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization studied in operations research. A set of cities is given to the salesman and he has to start from a city, visit all the cities only once and return back to the start city to complete a round tour such that the length of the tour is the shortest among all possible tours plans. As the TSP is a special class of Assignment Problem (AP) with additional constraints, a solution to the TSP, in general, is found using the methods available for solving APs. In the recent years considerable number of zeros assignment methods and ones assignment methods have been published by several researchers for solving APs.

The paper is organized as follows: In Section 1, brief introduction is given. The algorithm for the proposed ESAN-ESA method is presented in Section 2. In Section 3, two benchmark problems from the literature have been illustrated. Section 4 lists a set of 30 benchmark TSPs from the literature for testing the proposed algorithm. The results produced by the ESAN-ESA method on the 30 benchmark TSPs are compared with the already established optimal tour plans are shown in Section 5. Finally, in Section 6 conclusions are drawn.

Complexity: If n is the given number of cities to be visited, then the total number of possible routes (tours or trips) covering all cities can be given as a set of feasible solutions of the

TSP and is given as $(n-1)!$.
Length of a Tour: For a complete round tour or simply a tour to an n city TSP, the salesperson travels exactly n arcs (or n edges). The sum of the values (distances or costs) in every arc of a tour gives the length of the tour. The length of a tour may be in terms of kilometer or currency or time units.
Minimum Length Tour A tour: with the minimum length is called a minimum length tour or an optimal tour. The length of an optimal tour is denoted by the symbol Z. It is clear that, $Z \leq$ Length of a tour.
Lower Bound (LB): The sum of the minimum values in every row gives a lower bound (LB) for the value of the minimum length tour.

2. Algorithm for the Proposed ESAN-ESA Method

The term ESAN-ESA is coined from the names of the authors Murugesan and Esakkiammal. The ESAN-ESA method consists of two phases. In the first phase, a set of optimal subtours is generated using the zeros assignment technique based on the ME rules and in the second phase linking of all the subtours together to form a complete round tour is carried out based on the available 0-entry or 1-entry or the next higher entry among the upcoming unassigned cells. The algorithm is as follows:
Phase-I
(Generating subtours)
Step 1: Construct the Reduced Cost Matrix (RCM).

(a) **Perform the Row Minimum Subtraction (RMS) Operation.**
Subtract from each of the costs of every row of the TSP by its minimum cost. This will result in a reduced cost matrix, which will have at least one zero in each row. If each row and each column of the reduced cost matrix has at least one 0s, go to Step 2 to cover all the 0s entry; otherwise, go to Step 1(b).

(b) **Perform the Column Minimum Subtraction (CMS) Operation.**
Subtract from each of the entries of every column of the reduced cost matrix obtained in Step 0(a) by its minimum entry. These operations create at least one zero in each column. Go to Step 2 to cover all the 0s entry.

Step 2: Cover all the 0s with minimum number of lines using ME rules (The word ME is coined from the first letter of the names of the authors Murugesan and Esakkiammal):
Rule 1: To draw Minimum number of Lines to cover all 0s
Column-wise assignment

(i) Look at the columns successively from first to last until a column with exactly one unassigned 0-entry is found.

(ii) Make an assignment to this single 0-entry by creating a circle or a square around it.

(iii) Draw a horizontal line passing through that 0-entry.

(iv) Continue in this way until all the columns have been scrutinized.

Rule 2: To test the conditions for a complete assignment plan
Test whether the conditions for a complete assignment plan is achieved. If yes, write the assignment plan; otherwise, select the smallest element (say d_{ii}) out of those which do not lie on any of the drawn lines in the above matrix. Then subtract by d_{ii} from each uncovered element of the uncovered row(s) or column(s), which d_{ij} lies on it. This operation creates some new zero(s) to this row or column. Then, go to Rule 1.

If the conditions for a complete assignment plan are not satisfied through the above said two rues, then apply rule 3.

3. Numerical Illustrations

Phase-I (Generating Subtours)
Numerical Examples See Table 1.

4. Result Analysis

In order to measure the effectiveness of the proposed ESAN-ESA method, 30 benchmark problems, listed in Table 1, have been tested and the results are compared with the already established optimal solutions. The comparison of results is shown in Table 2.

From Table 2, we discover that out of 30 benchmark problems tested, for all 30 problems the proposed ESAN-ESA method has produced optimal tour plans directly. Also, it is observed that each of the TSP satisfies the bounding condition $TLB \leq Z \leq UB$.

5. Conclusion

In this paper, we have proposed a new method named ESAN-ESA to find the optimal tour plans to Traveling Salesman Problems. This method finds the optimal tour plan to a given TSP in two phases. In Phase-I, a set of optimal subtours is generated using the zeros assignment technique based on the ME rules. Formation of optimal tour plan, by linking neatly all the generated subtours, is carried out in Phase-II based on the existing least entry (that is, next 0-entry or 1-entry or the next higher entry) among the forthcoming unassigned cities. The proposed method has been implemented on 30 small size benchmark TSPs (symmetric, asymmetric and restricted categories) from the literature. Simulation results authenticate that the ESAN-ESA method is the best technique which generates optimal tour plans to all the 30 instances Hence, it is guaranteed that by applying the ESAN-ESA method one can generate an optimal tour plan to a given TSP.

Table 1: Subtours and the corresponding optimal tour plans generated by ESAN-ESA

Prob. No. #	Subtours generated by Phase-I	Length of Subtours	Optimal Tour(s) generated by Phase-II	Length of Tour (Z)
1.	$1 \to 3 \to 4 \to 2 \to 1$	22	$1 \to 3 \to 4 \to 2 \to 1$	22
2.	$1 \to 4 \to 1$	06	$1 \to 4 \to 2 \to 3 \to 1$;	19
	$2 \to 3 \to 2$	12	$4 \to 1 \to 3 \to 2 \to 4$	
3.	$1 \to 3 \to 1$	20	$2 \to 4 \to 3 \to 1 \to 2$;	
	$2 \to 4 \to 2$	16	$4 \to 2 \to 1 \to 3 \to 4$	42
4.	$1 \to 4 \to 1$	24	$2 \to 3 \to 4 \to 1 \to 2$;	
	$2 \to 3 \to 2$	16	$3 \to 2 \to 1 \to 4 \to 3$	46
5.	$1 \to 2 \to 4 \to 3 \to 1$	35	$1 \to 2 \to 4 \to 3 \to 1$	35
6.	$1 \to 5 \to 1$	02	$5 \to 1 \to 2 \to 3 \to 4 \to 5$	15
	$2 \to 3 \to 4 \to 2$	11		
7.	$1 \to 5 \to 1$	22	$5 \to 1 \to 2 \to 3 \to 4 \to 5$	65
	$2 \to 3 \to 4 \to 2$	41		
8.	$1 \to 5 \to 1$	20	$1 \to 5 \to 2 \to 3 \to 4 \to 1$	62
	$2 \to 3 \to 4 \to 2$	40		
9.	$1 \to 2 \to 4 \to 3 \to 5 \to 1$	26	$1 \to 2 \to 4 \to 3 \to 5 \to 1$	26
10.	$1 \to 4 \to 3 \to 1$	21		
	$2 \to 5 \to 2$	16	$2 \to 5 \to 3 \to 4 \to 1 \to 2$	47
11.	$1 \to 4 \to 2 \to 1$	07	$4 \to 1 \to 5 \to 3 \to 2 \to 4$	16
	$3 \to 5 \to 3$	08	$3 \to 5 \to 1 \to 4 \to 2 \to 3$	16
12.	$1 \to 3 \to 1$	16	$3 \to 1 \to 5 \to 2 \to 4 \to 3$	34
	$2 \to 4 \to 5 \to 2$	17		
	(Alternate subtours)			
	$1 \to 3 \to 1$	16	$1 \to 3 \to 4 \to 2 \to 5 \to 1$	34
	$2 \to 5 \to 4 \to 2$	17		
13.	$1 \to 5 \to 3 \to 1$	28		
	$2 \to 4 \to 2$	16	$4 \to 2 \to 1 \to 5 \to 3 \to 4$	46
14.	$1 \to 4 \to 2 \to 1$	13		
	$3 \to 5 \to 3$	12	$3 \to 5 \to 1 \to 4 \to 2 \to 3$	
15.	$1 \to 5 \to 1$	02	$5 \to 1 \to 2 \to 3 \to 4 \to 5$	26
	$2 \to 3 \to 4 \to 2$	11		
16.	$1 \to 3 \to 5 \to 2 \to 4 \to 1$	17	$1 \to 3 \to 5 \to 2 \to 4 \to 1$	17
17.	$1 \to 3 \to 1$	07	$1 \to 3 \to 4 \to 2 \to 5 \to 1$	16
	$2 \to 5 \to 4 \to 2$	08		
18.	$1 \to 2 \to 3 \to 1$	18		
	$4 \to 5 \to 4$	08	$4 \to 5 \to 3 \to 1 \to 2 \to 4$	29
19.	$1 \to 4 \to 3 \to 1$	20		
	$2 \to 5 \to 2$	20	$3 \to 1 \to 4 \to 5 \to 2 \to 3$	49
20.	$1 \to 3 \to 1$	04		
	$2 \to 4 \to 2$	02	$2 \to 4 \to 6 \to 5 \to 3 \to 1 \to 2$	44
	$5 \to 6 \to 5$	08		

Table 2: Comparison of results obtained by the ESAN-ESA with the optimal solutions

Prob. No. #	TLB	UB	Optimal length (Z)	Length by ESANESA	Prob. No. #	TLB	UB	Optimal length (Z)	Length by ESANESA
1.	22	26	22	22	16.	17	23	17	17
2.	18	20	19	19	17.	15	26	16	16
3.	36	46	42	42	18.	24	34	29	29
4.	40	52	46	46	19.	40	53	49	49
5.	36	42	35	35	20.	14	67	44	44
6.	13	26	15	15	21.	08	15	13	13
7.	62	73	65	65	22.	78	100	76	76
8.	60	80	62	62	23.	38	48	39	39
9.	26	33	26	26	24.	94	95	95	95
10.	37	57	47	47	25.	50	62	54	54
11.	16	20	16	16	26.	41	56	43	43
12.	33	38	34	34	27.	137	387	153	153
13.	44	53	46	46	28.	19	63	26	26
14.	25	30	26	26	29.	87	154	89	89
15.	13	26	42	42	30.	1540	4005	2056	2056

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