Fixed point of almost generalized $(\hat{\alpha}, \hat{\psi}, \hat{\varphi}, \hat{\theta})$-contractive type mappings in weak partial metric spaces

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This paper is dedicated to the occasion of Professor Gaston M. N’Guérékata’s 70th birthday

Abstract. This study introduces almost generalized $(\hat{\alpha}, \hat{\psi}, \hat{\varphi}, \hat{\theta})$-contractive type mappings and investigates some fixed point theorems for such mappings in weak partial metric spaces. Our results extend the implications of Altun and Durmaz [15] and other prior results in this area. Additionally, we present some examples that support our findings.

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1. Introduction and Background

Banach fixed point theorem has been expanded in numerous ways and it has undergone numerous generalisations in various metric spaces. Partial metric space (PMS), which Matthews [1] introduced in 1992, is a very intriguing generalisation of the metric space in which the self distance not required to be zero. By establishing a new class of contractive type mappings known as $\hat{\alpha} – \hat{\psi}$ contractive type mappings, Samet et al. [3] further expanded and generalised the Banach contraction principle. The $\hat{\alpha} – \hat{\psi}$ contractive type mappings were generalised by Karapinar and Samet[4]. On the other hand, Berinde [7, 8] introduced the concept of almost contractions in metric spaces. The concept of weak partial metric spaces, a generalisation of partial metric spaces, was first introduced by Heckmann [14] in 1999. Some results for mappings in weak partial metric spaces have recently been obtained in [17], [18],[19] and [20].

Definition 1.1. [12] Let $\Psi$ be the set of functions $\hat{\psi} : [0, \infty) \to [0, \infty)$ such that

(a) $\hat{\psi}$ is non decreasing and continuous;
(b) $\hat{\psi}(u) = 0 \iff u = 0$.

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Lemma 1.7. Let $\Gamma : W_p \rightarrow W_p$ and $\tilde{\alpha} : W_p \times W_p \rightarrow [0, \infty)$. $\Gamma$ is said to be $\tilde{\alpha}$-admissible if

$$\tilde{\alpha}(\eta_p, \zeta_p) \geq 1 \Rightarrow \tilde{\alpha}(\Gamma \eta_p, \Gamma \zeta_p) \geq 1$$

for all $\eta_p, \zeta_p \in W_p$.

Definition 1.3. Let $\Gamma : W_p \rightarrow W_p$ and $\tilde{\alpha} : W_p \times W_p \rightarrow [0, \infty)$ be two functions. Then $\Gamma$ is said to be triangular $\tilde{\alpha}$-admissible if $\Gamma$ is $\tilde{\alpha}$-admissible and for $\eta_p, \zeta_p, \delta_p \in W_p$, $\tilde{\alpha}(\eta_p, \delta_p) \geq 1$ and $\tilde{\alpha}(\delta_p, \zeta_p) \geq 1$ \Rightarrow $\tilde{\alpha}(\eta_p, \zeta_p) \geq 1$.

Lemma 1.4. Let $\Gamma : W_p \rightarrow W_p$ be a triangular $\tilde{\alpha}$-admissible mapping. Suppose that there exists $\eta_{p_0} \in W_p$ such that $\tilde{\alpha}(\eta_{p_0}, \Gamma \eta_{p_0}) \geq 1$. If we define a sequence $\{\eta_p\}$ by $\eta_{p,i+1} = \Gamma \eta_{p,i}$ for every $i \in \mathbb{N}_0$. Then we have $\tilde{\alpha}(\eta_{p_j}, \eta_{p_i}) \geq 1$ for all $j, i \in \mathbb{N}$ with $j > i$.

In 1992, Matthews [1] presented generalization of metric space as follows:

Definition 1.5. Let $W_p$ be a set which is non-empty. A mapping $\varrho : W_p \times W_p \rightarrow [0, \infty)$ is known as partial metric on $W_p$ if the following conditions are satisfied:

(PMS1) $\eta_p = \zeta_p \Leftrightarrow \varrho(\eta_p, \eta_p) = \varrho(\zeta_p, \zeta_p) = \varrho(\eta_p, \zeta_p)$;

(PMS2) $\varrho(\eta_p, \eta_p) \leq \varrho(\eta_p, \zeta_p)$;

(PMS3) $\varrho(\eta_p, \zeta_p) = \varrho(\zeta_p, \eta_p)$;

(PMS4) $\varrho(\eta_p, \zeta_p) \leq \varrho(\eta_p, \delta_p) + \varrho(\delta_p, \zeta_p) - \varrho(\delta_p, \delta_p)$ for all $\eta_p, \zeta_p, \delta_p \in W_p$.

Lemma 1.6. Let $(W_p, \varrho)$ be a partial metric space.

(a) A sequence $\{\eta_p\}$ in the space $(W_p, \varrho)$ converges to a point $\eta_p \in W_p$ if

$$\varrho(\eta_p, \eta_p) = \lim_{i \rightarrow \infty} \varrho(\eta_{p,i}, \eta_p),$$

(b) If $\lim_{i \rightarrow \infty} \varrho(\eta_{p,i}, \eta_{p,j})$ exists and finite then the sequence $\{\eta_{p,i}\}$ is a Cauchy sequence in space $(W_p, \varrho)$.

(c) If every Cauchy sequence $\{\eta_{p,i}\}$ in $W_p$ converges to a point $\eta_p \in W_p$, such that

$$\varrho(\eta_p, \eta_p) = \lim_{j \rightarrow \infty} \varrho(\eta_{p,j}, \eta_{p,i}) = \lim_{i \rightarrow \infty} \varrho(\eta_{p,i+1}, \eta_{p,i}) = \varrho(\eta_p, \eta_p)$$

Then $(W_p, \varrho)$ is complete.

Lemma 1.7. Let $\varrho$ be a partial metric on $W_p$, then the mapping $\varrho^m : W_p \times W_p \rightarrow \mathbb{R}^+$ such that

$$\varrho^m(\eta_p, \zeta_p) = \max\{\varrho(\eta_p, \zeta_p) - \varrho(\eta_p, \eta_p), \varrho(\eta_p, \zeta_p) - \varrho(\zeta_p, \zeta_p)\}$$

$$=\varrho(\eta_p, \zeta_p) - \min\{\varrho(\eta_p, \eta_p), \varrho(\zeta_p, \zeta_p)\}$$

is metric on $W_p$. Furthermore, $(W_p, \varrho^m)$ is metric space.

Let $(W_p, \varrho^m)$ be a partial metric space. Then

1. A sequence $\{\eta_{p,i}\}$ in $(W_p, \varrho^m)$ is a Cauchy sequence $\Leftrightarrow \{\eta_{p,i}\}$ is a Cauchy sequence in the metric space $(W_p, \varrho^m)$.

2. $(W_p, \varrho^m)$ is complete $\Leftrightarrow (W_p, \varrho)$ is complete. Moreover

$$\lim_{i \rightarrow \infty} \varrho^m(\eta_{p,i}, \eta_p) = 0 \Leftrightarrow \varrho(\eta_{p,i}, \eta_p) = \lim_{i \rightarrow \infty} \varrho(\eta_{p,i}, \eta_p) = \lim_{i,j \rightarrow \infty} \varrho(\eta_{p,i}, \eta_{p,j}).$$
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**Lemma 1.8.** \((18)\) Suppose that \(\{\eta_p\}\) be a sequence \(\eta_p \to \delta_p\) as \(i \to \infty\) in a partial metric space \((W_p, \varrho_p)\) such that \(\varrho_p(\delta_p, \delta_p) = 0\). Then \(\lim_{i \to \infty} \varrho_p(\eta_p, \zeta_p) = \varrho_p(\delta_p, \zeta_p)\) for every \(\zeta_p \in W_p\).

**Lemma 1.9.** \((18)\) Suppose that \(\{\eta_p\}\) be a sequence with \(\lim_{i \to \infty} \varrho_p(\eta_p, \eta_{p+1}) = 0\) such that \(\{\eta_p\}\) is not a Cauchy sequence in \((W_p, \varrho_p)\), and there exist two sequences \(\{i(u)\}\) and \(\{j(u)\}\) of positive integers such that \(i(u) > j(u) > u\), then following sequences

\[
\varrho_p(\eta_{p_{j(u)}}, \eta_{p_{j(u)+1}}), \varrho_p(\eta_{p_{i(u)}}, \eta_{p_{i(u)+1}}), \\
\varrho_p(\eta_{p_{j(u)-1}}, \eta_{p_{j(u)+1}}), \varrho_p(\eta_{p_{i(u)-1}}, \eta_{p_{i(u)+1}})
\]

\(\eta_p \to \infty\) when \(u \to \infty\).

**Lemma 1.10.** \((12), (16)\) Let \(W_p\) be a set which is non-empty. Suppose that \((W_p, \varrho_p)\) be a partial metric space.

1. If \(\eta_p \neq \zeta_p\) then \(\varrho_p(\eta_p, \zeta_p) > 0\),
2. If \(\varrho_p(\eta_p, \zeta_p) = 0\) then \(\eta_p = \zeta_p\).

By omitting the small self-distance axiom in partial metric spaces, Heckmann \([14]\) introduced the concept of weak partial metric space as follows:

**Definition 1.11.** \([14]\) Let \(W_p\) be a set which is non-empty. A mapping \(\varrho : W_p \times W_p \to [0, \infty)\) is known as weak partial metric on \(W_p\) if the following conditions are satisfied:

\((WPMS1)\) \(\eta_p = \zeta_p \iff \varrho_p(\eta_p, \eta_p) = \varrho_p(\zeta_p, \zeta_p) = \varrho_p(x, \zeta_p)\);

\((WPMS2)\) \(\varrho_p(\eta_p, \zeta_p) = \varrho_p(\zeta_p, \eta_p)\);

\((WPMS3)\) \(\varrho_p(\eta_p, \zeta_p) \leq \varrho_p(\eta_p, \delta_p) + \varrho_p(\delta_p, \zeta_p) - \varrho_p(\delta_p, \delta_p)\) for all \(\eta_p, \zeta_p, \delta_p \in W_p\).

and the pair \((W_p, \varrho_p)\) is called weak partial metric space (in short WPMS).

Additionally, Heckmann \([14]\) demonstrates that the weak small self-distance feature follows if \(\varrho_p\) is a weak partial metric on \(W_p\) i.e.

\[
\varrho_p(\eta_p, \zeta_p) \geq \frac{\varrho_p(\eta_p, \eta_p) + \varrho_p(\zeta_p, \zeta_p)}{2}
\]

for all \(\eta_p, \zeta_p \in W_p\).

Every partial metric space is obviously a weak partial metric space, but the converse may not be true. For example, for \(\eta_p, \zeta_p \in \mathbb{R}\) the function \(\varrho_p(\eta_p, \zeta_p) = \frac{e^{\eta_p} + e^{\zeta_p}}{2}\) is a weak partial metric space but not a partial metric on \(\mathbb{R}\).

**Lemma 1.12.** \([15]\) Let \((W_p, \varrho_p)\) be a weak partial metric space (WPMS).

(i) \(\{\eta_p\}\) is a Cauchy sequence in \((W_p, \varrho_p) \iff \) it is a Cauchy sequence in \((W_p, \varrho^m_p)\);

(ii) \((W_p, \varrho_p)\) is complete \(\iff (W_p, \varrho^m_p)\) is complete.

**Lemma 1.13.** \([17]\) Let \((W_p, \varrho_p)\) be a weak partial metric space and \(\{\eta_p\}\) is a sequence in \((W_p, \varrho_p)\). If \(\lim_{i \to \infty} \eta_p = \eta_p\) and \(\varrho_p(\eta_p, \eta_p) = 0\), then \(\lim_{i \to \infty} \varrho_p(\eta_p, \zeta_p) = \varrho_p(\eta_p, \zeta_p)\) for all \(\zeta_p \in W_p\).

**Definition 1.14.** \([13]\) Let \(\Psi\) be the set of all functions \(\hat{\psi} : [0, \infty) \to [0, \infty)\) satisfying the following conditions:

(i) \(\hat{\psi}(u) < \hat{\psi}(u)\) for all \(u > 0\)

(ii) \(\hat{\psi}(0) = 0\)

**Definition 1.15.** \([21]\) Let \(\Theta\) be the set of functions \(\hat{\vartheta} : [0, \infty) \to [0, \infty)\) such that

(i) \(\hat{\vartheta}\) is continuous;

(ii) \(\hat{\vartheta}(u) = 0 \iff u = 0\).

**Remark 1.16.** The convergence of sequences, Cauchy sequences, and completeness in a weak partial metric space are defined as being in a partial metric space.
2. Main Results

Definition 2.1. Let \((W_p, \mathcal{D}_p)\) be a weak partial metric space and \(\Gamma : W_p \to W_p\) be a given self map. We say that \(\Gamma\) is almost generalized \((\hat{\alpha}, \hat{\psi}, \hat{\phi}, \hat{\Theta})\)-contractive mapping if there exists \(\hat{\alpha} : W_p \times W_p \to [0, \infty)\) and \(\hat{\psi} \in \Psi\), \(\hat{\phi} \in \Phi\), \(\hat{\theta} \in \Theta\) and \(L \geq 0\) such that for all \(\eta_p, \zeta_p \in W_p\) we have

\[
\hat{\alpha}(\eta_p, \zeta_p)\hat{\psi}(\mathcal{D}_p(\Gamma\eta_p, \Gamma\zeta_p)) \leq \hat{\phi}(\mathcal{M}(\eta_p, \zeta_p)) + L\hat{\theta}(\mathcal{N}(\eta_p, \zeta_p))
\]

(2.1)

Where

\[
\mathcal{M}(\eta_p, \zeta_p) = \max \{\mathcal{D}_p(\eta_p, \zeta_p), \mathcal{D}_p(\eta_p, \Gamma\eta_p), \mathcal{D}_p(\zeta_p, \Gamma\zeta_p), \frac{1}{2}[\mathcal{D}_p(\eta_p, \Gamma\zeta_p) + \mathcal{D}_p(\zeta_p, \Gamma\eta_p)]\}
\]

(2.2)

and

\[
\mathcal{N}(\eta_p, \zeta_p) = \min \{\mathcal{D}_p^m(\eta_p, \Gamma\eta_p), \mathcal{D}_p^m(\zeta_p, \Gamma\eta_p)\}
\]

(2.3)

Theorem 2.2. Let \((W_p, \mathcal{D}_p)\) be a complete weak partial metric space and \(\Gamma : W_p \to W_p\) be self mapping. Suppose \(\hat{\alpha} : W_p \times W_p \to [0, \infty)\) be the mapping satisfying the conditions:

(i) \(\Gamma\) is triangular \(\hat{\alpha}\)-admissible;

(ii) \(\Gamma\) is almost generalized \((\hat{\alpha}, \hat{\psi}, \hat{\phi}, \hat{\Theta})\)-contractive mapping;

(iii) There exists \(\eta_{p_0} \in W_p\) such that \(\hat{\alpha}(\eta_{p_0}, \Gamma\eta_{p_0}) \geq 1\);

(iv) \(\Gamma\) is continuous.

Then \(\Gamma\) has a fixed point in \(W_p\).

Proof. Let there be an arbitrary point \(\eta_{p_0}\) such that \(\hat{\alpha}(\eta_{p_0}, \Gamma\eta_{p_0}) \geq 1\). Suppose there is a sequence \(\{\eta_{p_i}\}\) in \(W_p\) such that \(\eta_{p_{i+1}} = \Gamma\eta_{p_i}\) for all \(i \in \mathbb{N}_0\).

If \(\eta_{p_i} = \eta_{p_{i+1}}\) for some \(i \in \mathbb{N}_0\), then \(\eta_{p_i}\) is a fixed point of \(\Gamma\) and then proof of existence part of fixed point is finished. Suppose \(\eta_{p_i} \neq \eta_{p_{i+1}}\) for every \(i \in \mathbb{N}_0\). Then \(\mathcal{D}_p(\eta_{p_i}, \eta_{p_{i+1}}) > 0\) by Lemma 1.10. Now, since \(\Gamma\) is \(\hat{\alpha}\)-admissible, so

\[
\hat{\alpha}(\Gamma\eta_{p_{i}}, \eta_{p_{i+1}}) = \hat{\alpha}(\eta_{p_{i+1}}, \eta_{p_{i+2}}) \geq 1
\]

\[
\hat{\alpha}(\Gamma\eta_{p_{i+1}}, \eta_{p_{i+2}}) = \hat{\alpha}(\eta_{p_{i+2}}, \eta_{p_{i+3}}) \geq 1
\]

and using induction we have \(\hat{\alpha}(\eta_{p_i}, \eta_{p_{i+1}}) \geq 1\) for all \(i \in \mathbb{N}\).

Now, from (2.1) we have

\[
\hat{\psi}(\mathcal{D}_p(\eta_{p_i}, \eta_{p_{i+1}})) = \hat{\psi}(\mathcal{D}_p(\Gamma\eta_{p_{i-1}}, \Gamma\eta_{p_i})) \leq \hat{\phi}(\mathcal{M}(\eta_{p_{i-1}}, \eta_{p_i})) + L\hat{\theta}(\mathcal{N}(\eta_{p_{i-1}}, \eta_{p_i}))
\]

(2.4)

where

\[
\mathcal{N}(\eta_{p_{i-1}}, \eta_{p_i}) = \min \{\mathcal{D}_p^m(\eta_{p_{i-1}}, \Gamma\eta_{p_{i-1}}), \mathcal{D}_p^m(\eta_{p_i}, \Gamma\eta_{p_{i-1}})\}
\]

\[
= \min \{\mathcal{D}_p^m(\eta_{p_{i-1}}, \eta_{p_0}), \mathcal{D}_p^m(\eta_{p_i}, \eta_{p_0})\}
\]

(2.5)
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and

\[
\hat{\mathcal{M}}(\eta_{p_{i-1}}, \eta_{p_i}) = \max \left\{ \varrho(\eta_{p_{i-1}}, \eta_{p_i}), \varrho(\eta_{p_{i-1}}, \Gamma \eta_{p_{i-1}}), \varrho(\eta_{p_i}, \Gamma \eta_{p_{i-1}}), \frac{1}{2} \left[ \varrho(\eta_{p_{i-1}}, \Gamma \eta_{p_{i-1}}) + \varrho(\eta_{p_i}, \Gamma \eta_{p_{i-1}}) \right] \right\}
\]

\[
= \max \left\{ \varrho(\eta_{p_{i-1}}, \eta_{p_i}), \varrho(\eta_{p_{i-1}}, \eta_{p_{i+1}}), \varrho(\eta_{p_i}, \eta_{p_{i+1}}), \frac{1}{2} \left[ \varrho(\eta_{p_{i-1}}, \eta_{p_{i+1}}) + \varrho(\eta_{p_i}, \eta_{p_{i+1}}) \right] \right\}
\]

(2.6)

Now, using the condition (WPMS3) we have

\[
\varrho(\eta_{p_{i-1}}, \eta_{p_{i+1}}) \leq \varrho(\eta_{p_{i-1}}, \eta_{p_i}) + \varrho(\eta_{p_i}, \eta_{p_{i+1}}) - \varrho(\eta_{p_i}, \eta_{p_{i-1}})
\]

Therefore

\[
\frac{1}{2} [\varrho(\eta_{p_{i-1}}, \eta_{p_{i+1}}) + \varrho(\eta_{p_i}, \eta_{p_{i+1}})] \leq \frac{1}{2} [\varrho(\eta_{p_{i-1}}, \eta_{p_i}) + \varrho(\eta_{p_i}, \eta_{p_{i+1}}) - \varrho(\eta_{p_i}, \eta_{p_{i-1}})]
\]

\[
= \frac{1}{2} [\varrho(\eta_{p_{i-1}}, \eta_{p_i}) + \varrho(\eta_{p_i}, \eta_{p_{i+1}})]
\]

\[
\leq \max \{\varrho(\eta_{p_{i-1}}, \eta_{p_i}), \varrho(\eta_{p_i}, \eta_{p_{i+1}})\}
\]

(2.7)

By (2.6) and (2.7) we get that

\[
\hat{\mathcal{M}}(\eta_{p_{i-1}}, \eta_{p_i}) \leq \max \{\varrho(\eta_{p_{i-1}}, \eta_{p_i}), \varrho(\eta_{p_i}, \eta_{p_{i+1}})\}
\]

(2.8)

Now, using (2.5) and (2.8) in (2.4) and the fact that and \(\hat{\vartheta}(u) = 0 \iff u = 0\), we get that

\[
\hat{\varphi}(\varrho(\eta_{p_i}, \eta_{p_{i+1}})) \leq \varrho(\max \{\varrho(\eta_{p_i}, \eta_{p_{i+1}}), \varrho(\eta_{p_{i-1}}, \eta_{p_i})\})< \varrho(\eta_{p_{i-1}}, \eta_{p_i})
\]

(2.9)

Now, if \(\varrho(\eta_{p_i}, \eta_{p_{i+1}}) > \varrho(\eta_{p_{i-1}}, \eta_{p_i})\) using definition that \(\hat{\varphi}(u) < \hat{\varphi}(u)\) for \(u > 0\) we get

\[
\hat{\varphi}(\varrho(\eta_{p_i}, \eta_{p_{i+1}})) \leq \hat{\varphi}(\varrho(\eta_{p_{i-1}}, \eta_{p_i})) < \hat{\varphi}(\varrho(\eta_{p_{i-1}}, \eta_{p_i}))
\]

which is a contradiction. Hence

\[
\hat{\varphi}(\varrho(\eta_{p_i}, \eta_{p_{i+1}})) \leq \hat{\varphi}(\varrho(\eta_{p_{i-1}}, \eta_{p_i})) < \hat{\varphi}(\varrho(\eta_{p_{i-1}}, \eta_{p_i}))
\]

(2.10)

We get a sequence of non-negative real numbers \(\{\varrho(\eta_{p_i}, \eta_{p_{i+1}}) : i \in \mathbb{N}\}\) that decreases. Therefore there exists \(\lambda_0 \geq 0\) such that

\[
\lim_{i \to \infty} \varrho(\eta_{p_i}, \eta_{p_{i+1}}) = \lambda_0
\]

Let \(\lambda_0 > 0\). Then taking limit \(i \to \infty\) in (2.10) we get

\[
\hat{\varphi}(\lambda_0) \leq \hat{\varphi}(\lambda_0) < \hat{\varphi}(\lambda_0)
\]

This is contradiction. Hence

\[
\lim_{i \to \infty} \varrho(\eta_{p_i}, \eta_{p_{i+1}}) = 0
\]

(2.11)

We now show that \(\{\eta_{p_i}\}\) is a Cauchy sequence in \(W_p\), i.e. \(\lim_{i,j \to \infty} \varrho(\eta_{p_i}, \eta_{p_j}) = 0\). By contradiction, we prove it.

Let

\[
\lim_{i \to \infty} \varrho(\eta_{p_i}, \eta_{p_j}) \neq 0
\]
Then, with reference to lemma 1.9 all sequences tends to \( \mu_p > 0 \), when \( u \to \infty \). So we can see that

\[
\lim_{u \to \infty} \delta(p, \eta_{\pi_p(u)}, \eta_{\pi_p(u)}) = \mu_p \tag{2.12}
\]

Further corresponding to \( j(u) \), we can choose \( i(u) \) in such a way that it is smallest integer with \( i(u) > j(u) > u \). Then

\[
\lim_{u \to \infty} \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)}) = \mu_p \tag{2.13}
\]

Again,

\[
\delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)}) \leq \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)}) + \delta(p, \eta_{\pi_p(u)}, \eta_{\pi_p(u)-1}) - \delta(p, \eta_{\pi_p(u)}, \eta_{\pi_p(u)})
\]

Letting \( u \to \infty \) and using lemma 1.9 we get

\[
\lim_{u \to \infty} \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) = \mu_p \tag{2.14}
\]

Again note that

Now, since \( \Gamma \) is triangular \( \hat{\alpha} \)-admissible, from Lemma 1.4 we derive that \( \hat{\alpha}(\eta_p, \eta_p) \geq 1 \) for all \( i > j \in \mathbb{N}_0 \). Replacing \( \eta_p \) by \( \eta_{\pi_p(u)} \) and \( \zeta_p \) by \( \eta_{\pi_p(u)} \) in (2.1) respectively, we get

\[
\hat{\psi}(\delta(p, \eta_{\pi_p(u)}, \eta_{\pi_p(u)})) = \hat{\psi}(\delta(p, \Gamma \eta_{\pi_p(u)-1}, \Gamma \eta_{\pi_p(u)-1})) \leq \hat{\alpha}(\eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) \hat{\psi}(\delta(p, \Gamma \eta_{\pi_p(u)-1}, \Gamma \eta_{\pi_p(u)-1}))
\]

\[
\leq \hat{\psi}(\hat{\psi}(\delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}))) + L(\hat{\psi}(\hat{\psi}(\delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}))))
\]

Where

\[
\hat{M}(\eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) = \max \left\{ \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}), \delta(p, \eta_{\pi_p(u)-1}, \Gamma \eta_{\pi_p(u)-1}), \delta(p, \eta_{\pi_p(u)-1}, \Gamma \eta_{\pi_p(u)-1}), \frac{1}{2} [\delta(p, \eta_{\pi_p(u)-1}, \Gamma \eta_{\pi_p(u)-1}) + \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1})] \right\}
\]

\[
= \max \left\{ \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}), \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}), \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}), \frac{1}{2} [\delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) + \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)})] \right\}
\]

and

\[
\hat{N}(\eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) = \min \{ \delta(p, \eta_{\pi_p(u)-1}, \Gamma \eta_{\pi_p(u)-1}), \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) \}
\]

\[
= \min \{ \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}), \frac{1}{2} [\delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) + \delta(p, \eta_{\pi_p(u)-1}, \eta_{\pi_p(u)})] \} \tag{2.16}
\]

Letting \( u \to \infty \) in (2.17) and (2.16) and using (2.11), (2.12), (2.13), (2.14) and lemma 1.9 we get

\[
\lim_{u \to \infty} \hat{M}(\eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) = \max \{ \mu_p, 0, 0, \mu_p \} = \mu_p \tag{2.17}
\]

and

\[
\lim_{u \to \infty} \hat{N}(\eta_{\pi_p(u)-1}, \eta_{\pi_p(u)-1}) = 0. \tag{2.18}
\]
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Now Letting $u \to \infty$ in (2.15) and using (2.17) and (2.18) we get

$$\hat{\psi}(\mu_p) \leq \hat{\varphi}(\mu_p) < \hat{\psi}(\mu_p)$$

This is a contradiction, Therefore

$$\lim_{i,j \to \infty} \mathcal{D}_g(\eta_p, \eta_{p_i}) = 0 \quad (2.19)$$

This implies that $\{\eta_{p_i}\}$ is a Cauchy sequence in $(W_p, \mathcal{D}_g)$. On the other hand, since

$$\mathcal{D}_g^m(\eta_{p_i}, \eta_{p_j}) = \mathcal{D}_g(\eta_{p_i}, \eta_{p_j}) - \min\{\mathcal{D}_g(\eta_{p_i}, \eta_{p_j}), \mathcal{D}_g(\eta_{p_j}, \eta_{p_i})\} \leq \mathcal{D}_g(\eta_{p_i}, \eta_{p_j})$$

Again from Lemma 1.12 we have

$$\mathcal{D}_g(\eta_{p_i}, \eta_{p_j}) \leq \mathcal{D}_g(\eta_{p_{i+1}}, \eta_{p_{j+1}})$$

In the following, we omit the continuity assumption of $\Gamma$ in Theorem 2.2.

**Theorem 2.3.** Let $(W_p, \mathcal{D}_g)$ be a complete weak partial metric space and $\Gamma : W_p \to W_p$ be self mapping. Suppose $\hat{\alpha} : W_p \times W_p \to [0, \infty)$ be the mappings satisfying the conditions:

(i) $\Gamma$ is triangular $\hat{\alpha}$-admissible;

(ii) $\Gamma$ is almost generalized $(\hat{\alpha}, \hat{\psi}, \hat{\varphi}, \hat{\theta})$-contractive mapping;

(iii) There exists $\eta_{p_0} \in W_p$ such that $\hat{\alpha}(\eta_{p_0}, \Gamma \eta_{p_0}) \geq 1$;

(iv) If $\{\eta_{p_i}\}$ is a sequence in $W_p$ such that $\eta_{p_i} \to \eta_p \in W_p$, $\hat{\alpha}(\eta_{p_i}, \eta_{p_{i+1}}) \geq 1$ for all $i$, there exists a subsequence $\{\eta_{p_{i(u)}}\}$ of $\{\eta_{p_i}\}$ such that $\hat{\alpha}(\eta_{p_{i(u)}}, \eta_p) \geq 1$ for all $u$. Then $\Gamma$ has a fixed point in $W_p$. Further if $\delta_p, \delta_q$ are fixed points of $\Gamma$ such that $\hat{\alpha}(\delta_p, \delta_q) \geq 1$ then $\delta_p = \delta_q$.

**Proof.** From the proof of the Theorem 2.2, the sequence $\eta_{p_i}$ defined by $\eta_{p_{i+1}} = \Gamma \eta_{p_i}$ is Cauchy in $W_p$ and converges to $\delta_p \in W_p$. According to the assumptions, there is a subsequence of $\{\eta_{p_{i(u)}}\}$ of $\{\eta_{p_i}\}$ such that $\hat{\alpha}(\eta_{p_{i(u)}}, \delta_p) \geq 1$ for all $u$. We will now demonstrate that $\delta_p$ is a fixed point of $\Gamma$. Consider the alternative, then $\mathcal{D}_g(\delta_p, \Gamma \delta_p) > 0$.

Now in (2.1) replacing $\eta_p$ by $\eta_{p_{i(u)}}$ and $\zeta_p$ by $\delta_p$ we get

$$\delta_p = \lim_{i \to \infty} \eta_{p_{i+1}} = \lim_{i \to \infty} \Gamma \eta_{p_i} = \Gamma \delta_p$$

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Further, suppose \( \hat{\psi}(\partial \psi_{\eta_p(u)}, \Gamma \delta_p) = \hat{\psi}(\partial \psi_{\Gamma \eta_p(u)}, \Gamma \delta_p) \leq \hat{\alpha}(\eta_p(u), \delta_p) \hat{\psi}(\partial \psi_{\Gamma \eta_p(u)}, \Gamma \delta_p) \leq \hat{\phi}(\hat{\mathcal{M}}(\eta_p(u), \delta_p)) + L(\hat{\psi}(\hat{\mathcal{N}}(\eta_p(u), \delta_p))) \) (2.23)

Where
\[
\hat{\mathcal{M}}(\eta_p(u), \delta_p) = \max \left\{ \partial \psi_{\eta_p(u)}, \Gamma \delta_p, \partial \psi_{\eta_p(u)}, \Gamma \delta_p, \partial \psi_{\Gamma \eta_p(u)}, \Gamma \delta_p \right\}
\]
\[
= \max \left\{ \partial \psi_{\eta_p(u)}, \Gamma \delta_p, \partial \psi_{\eta_p(u)}, \Gamma \delta_p, \partial \psi_{\Gamma \eta_p(u)}, \Gamma \delta_p \right\}
\]
and
\[
\hat{\mathcal{N}}(\eta_p(u), \delta_p) = \min \{ \partial \psi_{\eta_p(u)}, \Gamma \delta_p, \partial \psi_{\Gamma \eta_p(u)} \}
\]
\[
= \min \{ \partial \psi_{\eta_p(u)}, \Gamma \delta_p, \partial \psi_{\Gamma \eta_p(u)} \}
\]
(2.25)

Now, taking \( u \to \infty \) in (2.24) and (2.25) and using the fact that due to (2.22) we have \( \partial \psi_{\delta_p, \delta_p} = 0 \), we get
\[
\lim_{u \to \infty} \hat{\mathcal{M}}(\eta_p(u), \delta_p) = \max \{ 0, 0, \partial \psi_{\delta_p, \Gamma \delta_p}, \frac{1}{2}[\partial \psi_{\delta_p, \Gamma \delta_p} + \partial \psi_{\delta_p, \Gamma \delta_p}] \} = \partial \psi_{\delta_p, \Gamma \delta_p}
\]
(2.26)

and
\[
\lim_{u \to \infty} \hat{\mathcal{N}}(\eta_p(u), \delta_p) = 0
\]
(2.27)

Now, taking \( u \to \infty \) in (2.23) and using (2.26), (2.27) and definitions of \( \hat{\psi}, \hat{\phi} \) and \( \hat{\theta} \) we get
\[
\hat{\psi}(\partial \psi_{\delta_p, \Gamma \delta_p}) = \hat{\psi}(\partial \psi_{\Gamma \delta_p, \Gamma \delta_p}) < \hat{\psi}(\partial \psi_{\delta_p, \Gamma \delta_p})
\]
which is a contradiction. Therefore \( \Gamma \delta_p = \delta_p \) i.e., \( \delta_p \) is a fixed point.

Further, suppose \( \delta_p \) and \( \delta_q \) be two fixed point of \( \Gamma \) such that \( \partial \psi_{\delta_p, \delta_q} > 0 \) and \( \hat{\alpha}(\delta_p, \delta_q) \geq 1 \) then replacing \( \eta_p \) by \( \delta_p \) and \( \zeta_p \) by \( \delta_q \) in (2.1) we get
\[
\hat{\psi}(\partial \psi_{\delta_p, \delta_q}) = \hat{\psi}(\partial \psi_{\Gamma \delta_p, \Gamma \delta_q}) \leq \hat{\alpha}(\delta_p, \delta_q) \partial \psi_{\Gamma \delta_p, \Gamma \delta_q}
\]
\[
\leq \hat{\phi}(\hat{\mathcal{M}}(\delta_p, \delta_q)) + L(\hat{\psi}(\hat{\mathcal{N}}(\delta_p, \delta_q)))
\]
(2.28)

Where
\[
\hat{\mathcal{M}}(\delta_p, \delta_q) = \max \left\{ \partial \psi_{\delta_p, \delta_q}, \partial \psi_{\delta_p, \Gamma \delta_p}, \partial \psi_{\delta_q, \Gamma \delta_q}, \frac{1}{2}[\partial \psi_{\delta_p, \Gamma \delta_q} + \partial \psi_{\delta_q, \Gamma \delta_p}] \right\}
\]
\[
= \max \left\{ \partial \psi_{\delta_p, \delta_q}, \partial \psi_{\delta_p, \Gamma \delta_p}, \partial \psi_{\delta_q, \Gamma \delta_q}, \frac{1}{2}[\partial \psi_{\delta_p, \Gamma \delta_q} + \partial \psi_{\delta_q, \Gamma \delta_p}] \right\}
\]
\[
= \max \left\{ \partial \psi_{\delta_p, \delta_q}, 0, 0, \frac{1}{2}[\partial \psi_{\delta_p, \delta_q} + \partial \psi_{\delta_p, \delta_q}] \right\} \text{by (WPMS2)}
\]
\[
= \partial \psi_{\delta_p, \delta_q}
\]
(2.29)
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and

\[
\hat{N}(\delta_p, \delta_q) = \min\{d^m_\varphi(\delta_p, \Gamma \delta_p), d^m_\varphi(\delta_q, \Gamma \delta_p)\}
\]

\[
= \min\{d^m_\varphi(\delta_p, \delta_p), d^m_\varphi(\delta_q, \delta_p)\}
\]

\[
= 0
\]  \hspace{1cm} (2.30)

By putting (2.29), (2.30) in (2.28) and using the definitions of \(\hat{\psi}, \hat{\varphi}\) and \(\hat{\vartheta}\) we get

\[
\hat{\psi}(d_\varphi(\delta_p, \delta_q)) \leq \hat{\varphi}(d_\varphi(\delta_p, \delta_q)) < \hat{\psi}(d_\varphi(\delta_p, \delta_q))
\]

This is contradictory. As a result, \(\Gamma\) has a unique fixed point. The evidence is now complete. ■

The theorems’ consequences are given below.

**Corollary 2.4.** Let \((W_p, d_\varphi)\) be a complete weak partial metric space. \(\Gamma : W_p \to W_p\) satisfy the criterion by self-mapping with

\[
\hat{\psi}(d_\varphi(\Gamma \eta_p, \Gamma \zeta_p)) \leq \hat{\varphi}(d_\varphi(\eta_p, \zeta_p)) + L(\hat{N}(\eta_p, \zeta_p))
\]  \hspace{1cm} (2.31)

For all \(\eta_p, \zeta_p \in W_p, \hat{\psi} \in \Psi, \hat{\varphi} \in \Phi\) and \(L \geq 0\). Then \(\Gamma\) has a unique fixed point in \(W_p\).

**Corollary 2.5.** Let \((W_p, d_\varphi)\) be a complete weak partial metric space. A self-mapping \(\Gamma : W_p \to W_p\) be such that

\[
d_\varphi(\Gamma \eta_p, \Gamma \zeta_p) \leq k(\hat{N}(\eta_p, \zeta_p))
\]

For all \(\eta_p, \zeta_p \in W_p, k \in (0, 1)\), where

\[
\hat{N}(\eta_p, \zeta_p) = \max\{d_\varphi(\eta_p, \zeta_p), d_\varphi(\eta_p, \Gamma \eta_p), d_\varphi(\zeta_p, \Gamma \zeta_p), \frac{1}{2}[d_\varphi(\eta_p, \Gamma \zeta_p) + d_\varphi(\zeta_p, \Gamma \eta_p)]\}
\]  \hspace{1cm} (2.32)

Then \(\Gamma\) has a unique fixed point in \(W_p\).

**Example 2.6.** Let \(W_p = [0, 1]\) and \(d_\varphi(\eta_p, \zeta_p) = \frac{1}{2}(\eta_p + \zeta_p)\). Then \(d^m_\varphi(\eta_p, \zeta_p) = \frac{1}{2}|\eta_p - \zeta_p|\). Therefore, since \((W_p, d^m_\varphi)\) is complete, the by Lemma 1.12 \((W_p, d_\varphi)\) is a complete weak partial metric space (WPMS).

Consider the mapping \(\Gamma : W_p \to W_p\) defined by \(\Gamma(\eta_p) = \frac{\eta_p}{3}\) and let \(\hat{\psi}, \hat{\varphi}, \hat{\vartheta} : [0, \infty) \to [0, \infty)\) be such that \(\hat{\psi}(u) = 2u, \hat{\varphi}(u) = \frac{2u}{3}\) and \(\hat{\vartheta}(u) = u\) for all \(u \geq 0\). If we define the functions \(\hat{\alpha} : W_p \times W_p \to [0, \infty)\) as

\[
\hat{\alpha}(\eta_p, \zeta_p) = \begin{cases} 
1 & \eta_p, \zeta_p \in [0, \frac{1}{2}] \\
0 & \eta_p, \zeta_p \in (\frac{1}{2}, 1] 
\end{cases}
\]  \hspace{1cm} (2.33)

We show that contractive condition of Theorem 2.2 is satisfied.

Let \(\eta_p, \zeta_p \in [0, \frac{1}{2}]\) we get

\[
\hat{\alpha}(\eta_p, \zeta_p)\hat{\psi}(d_\varphi(\Gamma \eta_p, \Gamma \zeta_p)) = \hat{\alpha}(\eta_p, \zeta_p)\hat{\psi}(d_\varphi(\eta_p, \zeta_p)) = \hat{\psi}(\frac{1}{2}(\frac{\eta_p + \zeta_p}{3})) = \frac{2}{3}d_\varphi(\eta_p, \zeta_p)
\]  \hspace{1cm} (2.34)
On the other side

\[
\hat{M}(\eta_p, \zeta_p) = \max \left\{ d_\varphi(\eta_p, \zeta_p), d_\varphi(\eta_p, \Gamma \eta_p), d_\varphi(\zeta_p, \Gamma \zeta_p), \frac{1}{2} \left[ d_\varphi(\eta_p, \Gamma \zeta_p) + d_\varphi(\zeta_p, \Gamma \eta_p) \right] \right\}
\]

\[
= \max \left\{ \frac{\eta_p + \zeta_p}{2}, \frac{2\eta_p}{3}, \frac{2\zeta_p}{3}, \frac{\eta_p + \zeta_p}{2} \right\}
\]

\[
= \frac{\eta_p + \zeta_p}{2} = d_\varphi(\eta_p, \zeta_p)
\]

(2.35)

and

\[
\hat{N}(\eta_p, \zeta_p) = \min \left\{ d_\varphi^m(\eta_p, \Gamma \zeta_p), d_\varphi^m(\zeta_p, \Gamma \eta_p) \right\}
\]

\[
= \min \left\{ d_\varphi^m(\eta_p, \frac{\eta_p}{3}), d_\varphi^m(\zeta_p, \frac{\eta_p}{3}) \right\}
\]

(2.36)

Therefore from (2.35) we get

\[
\hat{\varphi}(\hat{M}(\eta_p, \zeta_p)) + L(\hat{\varphi}(\hat{N}(\eta_p, \zeta_p))) = \hat{\varphi}\left(\frac{\eta_p + \zeta_p}{2}\right) + L\left(\frac{\eta_p + \zeta_p}{2}\right)
\]

\[
= \frac{2}{3} \frac{\eta_p + \zeta_p}{2} + L(\hat{N}(\eta_p, \zeta_p))
\]

\[
= \frac{2}{3} \left[ d_\varphi(\eta_p, \zeta_p) + L(\hat{N}(\eta_p, \zeta_p)) \right]
\]

(2.37)

Now since \( L(\hat{N}(\eta_p, \zeta_p)) = L(\min\{d_\varphi^m(\eta_p, \eta_p), d_\varphi^m(\zeta_p, \eta_p)\}) \geq 0 \) for all \( \eta_p, \zeta_p \in W_p \), and from (2.34) and (2.37) we get

\[
\frac{2}{3} d_\varphi(\eta_p, \zeta_p) \leq \frac{2}{3} \left[ d_\varphi(\eta_p, \zeta_p) + L(\hat{N}(\eta_p, \zeta_p)) \right]
\]

(2.38)

for all \( \eta_p, \zeta_p \in W_p \).

Now, let \( \eta_p, \zeta_p \in (\tfrac{1}{2}, 1] \), in this case the contractive conditions of theorem 2.2 is already satisfied since \( \hat{\varphi}(\eta_p, \zeta_p) = 0 \). It is clear that all the conditions of Theorem 2.2 hold. Hence \( \Gamma \) has a fixed point, which in this case is 0.

**Example 2.7.** Let \( W_p = [0, 1] \) and \( d_\varphi(\eta_p, \zeta_p) = \frac{1}{2} (\eta_p + \zeta_p) \). Then \( d_\varphi^m(\eta_p, \zeta_p) = \frac{1}{2} |\eta_p - \zeta_p| \). Therefore, since \((W_p, d_\varphi^m)\) is complete, the by lemma 1.12 \((W_p, d_\varphi)\) is a complete weak partial metric space (WPMS).

Consider the mapping \( \Gamma : W_p \rightarrow W_p \) defined by \( \Gamma(\eta_p)=\begin{cases} \eta_p^2 & \eta_p \in [0, \tfrac{1}{2}] \\ \eta_p & \eta_p \in (\tfrac{1}{2}, 1] \end{cases} \) and let \( \psi, \hat{\psi} : [0, \infty) \rightarrow [0, \infty) \)

be such that \( \psi(u) = u, \hat{\psi}(u) = \frac{u}{2} \) for all \( u \geq 0 \).

Now, we show that contractive condition of corollary 2.4 is satisfied for \( L = 1 \), i.e.,

\[
\hat{\psi}(d_\varphi(\Gamma \eta_p, \Gamma \zeta_p)) \leq \hat{\psi}(d_\varphi(\eta_p, \zeta_p)) + L(\hat{N}(\eta_p, \zeta_p))
\]

(2.39)

for all \( \eta_p, \zeta_p \in W_p \). Let \( \eta_p, \zeta_p \in [0, \tfrac{1}{2}] \), then

\[
\hat{\psi}(d_\varphi(\Gamma \eta_p, \Gamma \zeta_p)) = \hat{\psi}\left(\frac{\eta_p^2 + \zeta_p^2}{2}\right) = \frac{\eta_p^2 + \zeta_p^2}{2} \leq \frac{1}{2} \left( \frac{\eta_p + \zeta_p}{2} \right) = \frac{1}{2} d_\varphi(\eta_p, \zeta_p)
\]

\[
\leq \frac{1}{2} d_\varphi(\eta_p, \zeta_p) + \min\{d_\varphi^m(\eta_p, \Gamma \eta_p), d_\varphi^m(\zeta_p, \Gamma \eta_p)\}
\]

\[
= \hat{\psi}(d_\varphi(\eta_p, \zeta_p)) + \min\{d_\varphi^m(\eta_p, \Gamma \eta_p), d_\varphi^m(\zeta_p, \Gamma \eta_p)\}
\]
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Now, let \(\eta_p, \zeta_p \in (\frac{1}{2}, 1]\), then result is clear since in this case \(d_\varrho(\Gamma \eta_p, \Gamma \zeta_p) = 0\). As a result, all requirements of corollary 2.4 are completely satisfied. As a result, it has a fixed point, which in this instance is 0.

Now, we demonstrate that the contractive requirement of Corollary 2.5 is met.

**Example 2.8.** Let \(W_p = [0, 1]\) and \(d_\varrho(\eta_p, \zeta_p) = \frac{1}{2}(\eta_p + \zeta_p)\). Then \(d_\varrho^m(\eta_p, \zeta_p) = \frac{1}{2}|\eta_p - \zeta_p|\). Therefore, since \((W_p, d_\varrho)\) is complete, the by lemma 1.12 \((W_p, d_\varrho)\) is a complete weak partial metric space (WPMS).

Consider the mapping \(\Gamma : W_p \rightarrow W_p\) defined by \(\Gamma(\eta_p) = \eta_p^3\). Then

\[
d_\varrho(\Gamma \eta_p, \Gamma \zeta_p) = d_\varrho(\eta_p, \zeta_p) = \frac{1}{3}d_\varrho(\eta_p, \zeta_p)
\]

On the other hand side

\[
\hat{M}(\eta_p, \zeta_p) = \max \{d_\varrho(\eta_p, \zeta_p), d_\varrho(\eta_p, \Gamma \eta_p), d_\varrho(\zeta_p, \Gamma \zeta_p), \frac{1}{2}[d_\varrho(\eta_p, \zeta_p) + d_\varrho(\zeta_p, \Gamma \eta_p)]\}
\]

\[
= \max \{d_\varrho(\eta_p, \zeta_p), d_\varrho(\eta_p, \frac{2\eta_p + \zeta_p}{3}), d_\varrho(\zeta_p, \frac{2\zeta_p + \eta_p}{3}), \frac{1}{2}[d_\varrho(\eta_p, \zeta_p) + d_\varrho(\zeta_p, \eta_p)]\}
\]

\[
\hat{M}(\eta_p, \zeta_p) = \frac{\eta_p + \zeta_p}{2}
\]

From (2.40) and (2.41) we get

\[
\frac{1}{3}d_\varrho(\eta_p, \zeta_p) \leq k d_\varrho(\eta_p, \zeta_p)
\]

for \(k \in [\frac{1}{3}, 1]\), i.e.

\[
d_\varrho(\Gamma \eta_p, \Gamma \zeta_p) \leq k(\hat{M}(\eta_p, \zeta_p))
\]

for \(k \in [\frac{1}{3}, 1]\). It is evident from (2.42) that it satisfies the requirement of Corollary 2.5. As a result, it has a fixed point, which in this instance is 0.

3. Conclusion

In this study, we proved certain fixed point theorems in the context of complete weak partial metric spaces using triangular \(\hat{\alpha}\)-admissible mappings and provided some implications of the main findings. We included some examples to support our results. The results in this article expand upon and generalise several results from the existing literature.

References


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