Abstract
In this paper we introduce $\alpha$-stable local necks, $\alpha$-stable monogenically directable, $\alpha$-stable monogenically strongly directable, $\alpha$-stable monogenically trap directable, $\alpha$-stable uniformly monogenically directable, $\alpha$-stable uniformly monogenically strongly directable, $\alpha$-stable uniformly monogenically trap-directable fuzzy automata. We have shown that $\alpha$-stable local necks of fuzzy automaton exists then it is $\alpha$-stable subautomaton. Further we prove some equivalent conditions on fuzzy automaton.

Keywords
$\alpha$-Stable local necks, $\alpha$-Stable monogenically directable, $\alpha$-Stable monogenically trap-directable, $\alpha$-Stable uniformly monogenically trap-directable.

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1. Introduction
Fuzzy set was introduced by L. A. Zadeh in 1965 [8]. The fuzzy set is a simple mathematical tool for representing the inevitability of vagueness, uncertainty, and imprecision in everyday life. W.G. Wee extended the fuzzy idea to automata in 1967 [7]. Later, numerous academics adapted the fuzzy notion to a wide range of domains, and it has a wide range of applications. J.N. Mordeson and D. S. Malik gave a detailed account of fuzzy automata and languages in their book [6].

T. Petkovic et al. [1] discussed directable automata, monogenically directable, generalized directable using necks. T. Petkovic et al.[3] introduce and studied trap-directable, trapped automata and other related automata. Also, we refer the survey paper Directable automata and their generalizations were investigated by S. Bogdanovic et al [2]. Further the necks and local necks of fuzzy automata were studied and discussed in [4, 5]. In this paper we introduce $\alpha$-stable local necks, $\alpha$-stable monogenically directable, $\alpha$-stable monogenically strongly directable, $\alpha$-stable monogenically trap directable, $\alpha$-stable uniformly monogenically directable, $\alpha$-stable uniformly monogenically strongly directable, $\alpha$-stable uniformly monogenically trap-directable fuzzy automata. We have shown that $\alpha$-stable local necks of fuzzy automata exists then it is $\alpha$-stable subautomata. Further we prove some equivalent conditions on fuzzy automaton.

2. Preliminaries

Definition 2.1. [6] A fuzzy automaton $S = (D, I, \psi)$, where,

$D$ - set of states $\{d_0, d_1, d_2, \ldots , d_n\}$,
$\textbf{I}$ - alphabets (or) input symbols,
$\psi$ - function from $D \times I \times D \rightarrow [0, 1]$,

The set of all words of $I$ is denoted by $I^*$. The empty word is denoted by $\lambda$, and the length of each $t \in I^*$ is denoted by $|t|$.

Definition 2.2. [6] Let $S = (D, I, \psi)$ be a fuzzy automaton. The extended transition function is defined by $\psi^*: D \times I^* \times D \rightarrow [0, 1]$ and is given by
\[ \psi^*(d_i, t, d_j) = \begin{cases} 1 & \text{if } d_i = d_j \\ 0 & \text{if } d_i \neq d_j \end{cases} \]

\[ \psi^*(d_i, t', d_j) = \bigvee_{d_i \in D} \{ \psi^*(d_i, t, d_i) \wedge \psi(d_i, t', d_j) \}, t \in I^*, t' \in I. \]

**Definition 2.3.** [4] Let \( S = (D, I, \psi) \) be a fuzzy automaton. Let \( D' \subseteq D \). Let \( \psi' \) be the restriction of \( \psi \) and let \( S' = (D', I, \psi') \). The fuzzy automaton \( S' \) is called a subautomaton of \( S \) if

(i) \( \psi : D' \times I \times D' \rightarrow [0,1] \)

(ii) For any \( d_i \in D' \) and \( \psi(d_i, t, d_j) > 0 \) for some \( t \in I^* \), then \( d_j \in D' \).

**Definition 2.4.** [6] Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is said to be strongly connected if for every \( d_i, d_j \in D \), there exists \( t \in I^* \) such that \( \psi^*(d_i, t, d_j) > 0 \). Equivalently, \( S \) is strongly connected if it has no proper subautomaton.

**Definition 2.5.** [4] Let \( S = (D, I, \psi) \) be a fuzzy automaton. A state \( d_i \in D \) is called a neck of \( S \) if there exists \( t \in I^* \) such that \( \psi^*(d_i, t, d_j) > 0 \) for every \( d_j \in D \).

In that case \( d_i \) is also called \( t \)-neck of \( S \) and the word \( t \) is called a directing word of \( S \).

If \( S \) has a directing word, then we say that \( S \) is a directable fuzzy automaton.

**Definition 2.6.** [5] Let \( S = (D, I, \psi) \) be a fuzzy automaton. If \( d_i \in Q \) is called local neck of \( S \), if it is neck of some directable subautomaton of \( S \). The set of all local necks of \( S \) is denoted by \( LN(S) \).

### 3. \( \alpha \)-Stable Local Neck of Fuzzy Automata

**Definition 3.1.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. If \( S \) is said to be \( \alpha \)-stable fuzzy automaton then \( \psi(d_i, t, d_j) \geq \alpha > 0, \forall t \in I, \alpha = \text{Fixed value in } [0,1] \).

**Definition 3.2.** Let \( S = (D, I, \psi) \) be a fuzzy automaton and let \( d_i \in D \). The \( \alpha \)-stable subautomaton of \( S \) generated by \( d_i \) is denoted by \( \langle d_i \rangle \).

It is given by \( \langle d_i \rangle = \{ d_j \mid \psi^*(d_i, t, d_j) \geq \alpha > 0, t \in I^* \} \).

If it exists, then it is called the \( \alpha \)-stable least subautomaton of \( S \) containing \( d_i \).

**Definition 3.3.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. For any non-empty \( D' \subseteq D \), the \( \alpha \)-stable subautomaton of \( S \) generated by \( d_i \in D' \) is denoted by \( \langle d_i \rangle \).

It is given by \( \langle d_i \rangle = \{ d_j \mid \psi^*(d_i, t, d_j) \geq \alpha > 0, d_j \in D', t \in I^* \} \).

This is called the \( \alpha \)-stable least subautomaton of \( S \) containing \( d_i \).

**Definition 3.4.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. A state \( d_i \in D \) is called \( \alpha \)-stable local neck of \( S \) if it is \( \alpha \)-stable neck of some \( \alpha \)-stable directable subautomaton of \( S \). The set of all \( \alpha \)-stable local necks of \( S \) is denoted by \( \alpha \text{SLN}(S) \).

**Definition 3.5.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is called \( \alpha \)-stable monogenically directable if every monogenic subautomaton of \( S \) is \( \alpha \)-stable directable.

**Definition 3.6.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is called \( \alpha \)-stable monogenically strongly directable if every monogenic subautomaton of \( M \) is \( \alpha \)-stable strongly directable.

**Definition 3.7.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is said \( \alpha \)-stable monogenically trap-directable if every monogenic subautomaton of \( S \) has a single \( \alpha \)-stable neck.

**Definition 3.8.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. If \( t \in I^* \) is \( \alpha \)-stable common directing word of \( S \) if \( t \) is a \( \alpha \)-stable directing word of every monogenic subautomaton of \( S \). The set all \( \alpha \)-stable common directing words of \( S \) will be denoted by \( \alpha \text{SCDW}(S) \).

In other words, \( \alpha \text{SCDW}(S) = \cap_{d_i \in D} \alpha \text{CDW}((d_i)) \).

**Definition 3.9.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is called \( \alpha \)-stable uniformly monogenically directlyable fuzzy automaton if every monogenic subautomaton of \( S \) is \( \alpha \)-stable directable and have atleast one \( \beta \)-weak common directing word.

**Definition 3.10.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is called \( \alpha \)-stable uniformly monogenically strongly directlyable fuzzy automaton if every monogenic subautomaton of \( S \) is strongly \( \alpha \)-stable directable and have atleast one \( \alpha \)-stable common directing word.

**Definition 3.11.** Let \( S = (D, I, \psi) \) be a fuzzy automaton. \( S \) is called \( \alpha \)-stable uniformly monogenically trap-directable fuzzy automaton if every monogenic subautomaton of \( S \) has a single \( \alpha \)-stable neck and have atleast one \( \alpha \)-stable common directing word.

### 4. Properties of \( \alpha \)-Stable Local Neck of Fuzzy Automata

**Theorem 4.1.** Let \( S = (D, I, \psi) \) be a fuzzy automaton and \( d_i \in D \). Then the following conditions are equivalent:

(i) \( d_i \) is a \( \alpha \)-stable local neck;

(ii) \( (d_i) \) is a strongly \( \alpha \)-stable directable fuzzy automaton;

(iii) For every \( t' \in I^* \), there exists \( t \in I^* \) such that \( \psi^*(d_i, t, d_j) \geq \alpha > 0 \).

**Proof.** (i) \( \Rightarrow \) (ii)

Let \( d_i \) be a \( \alpha \)-stable local neck of \( S \). Then there exists a \( \alpha \)-stable directable subautomaton \( S' \) of \( S \) such that \( d_i \in \alpha \text{SN}(S') \). Thus \( \alpha \text{SN}(S') \) is a \( \alpha \)-stable directlyable fuzzy automaton. Also, \( (d_i) \subseteq \alpha \text{SN}(S') \), and \( \alpha \text{SN}(S') \) is strongly connected, then \( (d_i) = \alpha \text{SN}(S') \). Therefore, \( (d_i) \) is a
strongly $\alpha$-stable directable fuzzy automaton.

$\forall i \Rightarrow (iii)$

Let $\langle d_i \rangle$ be a strongly $\alpha$-stable directable fuzzy automaton. Then $d_i$ is a $t$-$\alpha$-stable neck of $\langle d_i \rangle$ for some $t \in I^*$. Since $\langle d_i \rangle$ is strongly $\alpha$-stable directable, for every $i' \in I^*$, there exists some $d_{i'} \in \langle d_i \rangle$ such that $\{\psi^*(d_i, i', d_i) \geq \alpha \} > 0$. Now, $\psi^*(d_i, i', d_i) = \{\bigwedge_{d_i \in D} \{\psi^*(d_i, i', d_i), \psi^*(d_i, i, d_i)\} \} \geq \alpha > 0$.

$(iii) \Rightarrow (i)$

(iii) clearly shows that $d_i$ is a $t-\alpha$-stable neck of $\langle d_i \rangle$, and hence, it is a $\alpha$-stable local neck of $S$.

**Theorem 4.2.** Let $S = (D, I, \psi)$ be a fuzzy automaton. If $\alpha SLN(S) \neq \emptyset$, then $\alpha SLN(S)$ is a $\alpha$-stable subautomaton of $S$.

**Proof.** Let $d_i \in \alpha SLN(S)$ and $t \in I$. Then, the monogenic $\alpha$-stable subautomaton $\langle d_i \rangle$ of $S$ is strongly $\alpha$-stable directable. Now, $\langle d_i \rangle \subseteq \langle d_i \rangle$, for some $d_i \in \langle d_i \rangle$. Since $\langle d_i \rangle$ is strongly connected, $\langle d_i \rangle = \langle d_i \rangle$. Therefore, $d_i$ is also a $\alpha$-stable local neck of $S$, i.e., $d_i \in \alpha SLN(S)$. Hence, $\alpha SLN(S)$ is a $\alpha$-stable subautomaton of $S$.

**Theorem 4.3.** Let $S = (D, I, \psi)$ be a fuzzy automaton. Then the following conditions are equivalent:

(i) Every state of $D$ in $S$ is a $\alpha$-stable local neck;

(ii) $S$ is $\alpha$-stable monogenically strongly directable;

(iii) $S$ is $\alpha$-stable monogenically directable and $\alpha$-stable reversible;

(iv) $S$ is a direct sum of $\alpha$-stable strongly directable fuzzy automata;

(v) $\forall d_i \in D (\exists t \in I^*) \forall i' \in I^*$ such that $\psi^*(d_i, i', d_i) \geq \alpha > 0$.

**Proof.** (i) $\Rightarrow$ (ii)

If every state $d_i \in D$ is a $\alpha$-stable local neck of $S$. Then we have that for every $d_i \in D$ the $\alpha$-stable monogenic subautomaton $\langle d_i \rangle$ of $D$ in $S$ is $\alpha$-stable strongly directable. Hence, $S$ is $\alpha$-stable monogenically strongly directable.

(ii) $\Rightarrow$ (iii)

If $S$ is $\alpha$-stable monogenically strongly directable, then it is $\alpha$-stable monogenically directable. Now, every $\alpha$-stable monogenic subautomaton of $S$ is strongly connected, hence $S$ is $\alpha$-stable reversible.

(iii) $\Rightarrow$ (iv)

If $S$ is $\alpha$-stable reversible, then it is a direct sum of $\alpha$-stable strongly connected fuzzy automata $S_\beta$, $\beta \in Y$. Let $\beta \in Y$ and $d_i \in D_\beta$. Then $\langle d_i \rangle = S_\beta$. Since $S_\beta$ is strongly connected, and by the $\alpha$-stable monogenic directability of $S$ we have that $S_\beta = \langle d_i \rangle$ is $\alpha$-stable directable. Therefore, $S_\beta$ is $\alpha$-stable strongly directable, for any $\beta \in Y$.

(iv) $\Rightarrow$ (i)

Let $S$ be a direct sum of $\alpha$-stable strongly directable fuzzy automata $S_\beta$, $\beta \in Y$. Then for each state $d_i \in D$, there exists $\beta \in Y$ such that $d_i \in D_\beta$, that is, $d_i \in S_\beta$ is $\alpha$-stable, so $d_i$ is a $\alpha$-stable local neck of $S$.

(i) $\Rightarrow$ (v)

Since every state of $S$ is a $\alpha$-stable local neck, for any $d_i \in D$, $\langle d_i \rangle$ is $\alpha$-stable monogenically strongly directable. Hence, $\langle d_i \rangle$ is $\alpha$-stable reversible.

(v) $\Rightarrow$ (i)

This is an immediate consequence of proof of the Theorem 4.1.

### 5. Conclusion

We introduce $\alpha$-stable local necks, $\alpha$-stable monogenically directable, $\alpha$-stable monogenically strongly directable, $\alpha$-stable monogenically trap-directable, $\alpha$-stable uniformly monogenically directable, $\alpha$-stable uniformly monogenically strongly directable, $\alpha$-stable uniformly monogenically trap-directable fuzzy automata. We have shown that $\alpha$-stable local necks of fuzzy automata exists then it is $\alpha$-stable subautomata. Further we prove a some equivalent conditions on fuzzy automaton.

### References


